

國立中央大學 108 學年度碩士班考試入學試題

所別： 數學系碩士班 應用數學組 (一般生)

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數學系碩士班 應用數學組 (在職生)

科目： 微積分

本科考試禁用計算器

Problem 1. (12%) Find $\lim_{x \rightarrow 1^-} \left(\frac{\pi}{2} - \arcsin x \right)^{\arctan \frac{\sqrt{1-x^2}}{x}}$.

Problem 2. (12%) Find $\frac{d}{dx} \int_{\arccos x}^{\exp(x^2)} \frac{u^2}{4+u^4} du$ for $-1 < x < 1$.

Problem 3. (12%) Evaluate $\int_0^1 x^2 \arctan x dx$ using integration by parts with $u = x^2$ and $dv = \arctan x dx$.

Problem 4. (12%) Find the indefinite integral $\int \frac{dx}{x^n(x-1)}$, where n is a positive integer.

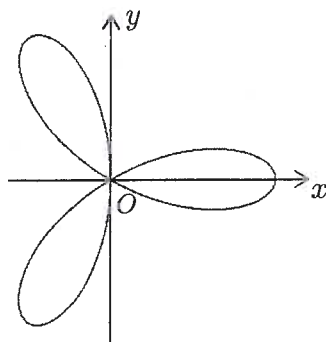
Problem 5. (12%) Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{3+2\cos(4x)}$.

Problem 6. (10%) Find the Maclaurin series of the function $y = \frac{\arctan x}{1-x}$. Note that you need to specify the interval of convergence of the Maclaurin series.

Problem 7. (10%) Find the maximum of $x^2 + y^2 + z^2$ subject to the constraints $x - y - 1 = y^2 - z^2 = 0$.

Problem 8. (10%) Let T be the trapezoid with vertices $(1, 1)$, $(2, 2)$, $(2, 0)$ and $(4, 0)$. Evaluate the double integral $\iint_T e^{(y-x)/(y+x)} dA$ by transforming to polar coordinates.

Problem 9. (10%) Let C be the polar curve with polar representation $r = \cos 3\theta$, $0 \leq \theta \leq \pi$ (see the Figure below).



Find the area enclosed by the curve.

參考用