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科目： 線性代數

本科考試禁用計算器

*請在答案卷內作答

參考用

(20%) 1. Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \\ 1 & 4 & 2 & 3 \\ 1 & 3 & 3 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and $c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

(a) Find the reduced row echelon form of A and the rank of A. (5%)
(b) Find the inverse of A if it exists. (5%)
(c) Find the set of solutions of the linear system $AX = b$. (5%)
(d) Find the set of solutions of the linear system $AX = c$. (5%)

(30%) 2. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$.

(a) Determine the characteristic polynomial of A and find the eigenvalues of A. (6%)
(b) For each eigenvalue λ of A, find the eigenspace corresponding to λ. (8%)
(c) Determine whether A is diagonalizable. Explain why. (4%)
(d) Determine the Jordan canonical form of A. (4%)
(e) Compute A^{101}. (8%)

(20%) 3. Let V be the space of functions from $\mathbb{R}$ to $\mathbb{R}$ and let U be the subset consisting of continuous functions in V. Let $S = \{1, \sin x, \cos x\}$ and $W = Span(S)$. Define $T : W \to W$ by $T(f) = f'$, where f' is the derivative of f.
(a) Prove or disprove that U is a subspace of V. (4%)
(b) Show that S is a basis for W. (4%)
(c) Show that T is a linear transformation and find the null space of T. (4%)
(d) Find the matrix representation of T in the ordered basis S. (4%)
(e) For each eigenvalue λ of A, find the set of eigenvectors corresponding to λ. (4%)

(15%) 4. (a) State the Dimension Theorem. (5%)
(b) Let $A, B \in M_{n\times n}(\mathbb{R})$. Prove or disprove that $\text{rank}(A) \geq \text{rank}(AB)$. (5%)
(c) Let $A, B \in M_{n\times n}(\mathbb{R})$. Prove or disprove that $\text{rank}(B) \geq \text{rank}(AB)$. (5%)

(15%) 5. Let $V = \mathbb{R}^4$, $S = \{(0,0,1,1), (0,1,0,1), (0,1,1,0)\}$ and $W = Span(S)$.
(a) Find an orthogonal basis for W. (8%)
(b) Determine the dimension of the orthogonal complement of W. (3%)
(c) Find an orthogonal basis for the orthogonal complement of W. (4%)