國立中央大學99學年度碩士班考試入學試題卷

所別：經濟學系碩士班 不分組(一般生)　科目：統計學　共 4 頁 第 1 頁

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

請在答案紙上按題號順序做答，否則不予計分

最後一頁為附表，以供查表計算所需

1. (20分) X 和 Y 為隨機變數，且 X 的平均數為 μ_X，變異數為 4，而 Y 的均數為 0，變異數為 σ_Y^2。若我們已知 $E(X|Y)=2Y^2$，$E(Y|X)=-3+0.5X$。請計算 μ_X，σ_Y^2，以及 $\text{cov}(X,Y)$。若我們得知 $\text{var}(Y|X)$ 為一實數 C，請計算 $\text{var}(Y|X)$。

2. (10分) $\{X_1,\ldots,X_n\}$ 為一組具有有限變異數 σ_0^2 的 i.i.d.隨機變數，其均數 μ_0 為已知。令

$$\hat{\sigma}_n^2=\frac{(X_1-\mu_0)}{2}+\frac{(X_2-\mu_0)}{3}+\frac{(X_{n-1}-\mu_0)}{12}+\frac{(X_n-\mu_0)}{12}$$

請問 $\hat{\sigma}_n^2$ 是否為 σ_0^2 的不偏估計式？是否為 σ_0^2 的一致估計式？

3. (10分) 現有一包含49戶台北市家庭每月飲食支出的隨機樣本，其樣本平均數為12,800元，樣本標準差為3,500元。如果台北市家庭每月飲食支出為常態分配，請問在95%信賴係數下，平均數的信賴區間為何？如果我們不能確定飲食是否為常態分配，請問在95%信賴係數下，平均數大致的信賴區間為何？

4. (10分) 參加校慶活動的學生中，大一、大二、大三、大四的學生各佔四分之一，其中大一學生中的百分之四十，大二學生中的百分之三十，大三學生中的百分之三十，大四學生中的百分之二十為女生。如今隨機抽出一名學生參加校長聚會。如果抽出的為女學生，則其為大一學生的機率為何？如果抽出的為男學生，則其為大三學生的機率為何？

5. (每小題4分，共16分) Multiple-Choice Questions

5-1. In a test of $H_0:\mu=9$ vs. $H_1:\mu\neq 9$, a sample of size 250 leads to a p-value of 0.034. Which of the following must be true?

(a) A 95% confidence interval for μ calculated from these data will not include $\mu=9$.

(b) At the 5% level if H_0 is rejected, the probability of a Type II error is 0.034.

(c) The 95% confidence interval for μ calculated from these data will be centered at $\mu=9$.

(d) The null hypothesis should not be rejected at the 5% level.

(e) The sample size is insufficient to draw a conclusion with 95% confidence.

5-2 The following information was obtained from independent random samples. Assume normally distributed populations with equal variances.

	Sample 1	**Sample 2**
Sample Mean	45	42
Sample Variance	85	90
Sample Size	10	12

The 95% confidence interval for the difference between the two population means is

(a) -5.372 to 11.372　(b) -5 to 3　(c) - 4.86 to 10.86　(d) -3.52 to 9.12　(e) -2.65 to 8.65

5-3. An investigator was studying a territorial species of Taiwanese ants. Forty-nine ant pairs were randomly selected; both members of each of these pairs were from the same colony. Fifty-five additional ant pairs were randomly selected; the two members in each of these pairs were from different colonies. The pairs were placed in a place and observed to see whether they exhibited aggressive behavior. The results are shown in the table below.

	Aggressive	Nonaggressive	Total
Same colony	40 (33.5)	9 (15.5)	49
Different colonies	31 (37.5)	24 (17.5)	55
Total	71	33	104

A Chi-square test for homogeneity was conducted, resulting in $\chi^2 = 7.638$. The expected counts are shown in parentheses in the table. Which of the following sets of statements follows from these results?

(a) χ^2 is not significant at the 0.05 level.

(b) χ^2 is significant, $0.01 < p < 0.05$; the counts in the table suggest that ant pairs from the same colony are less likely to be aggressive than ant pairs from different colonies.

(c) χ^2 is significant, $0.01 < p < 0.05$; the counts in the table suggest that ant pairs from different colonies are less likely to be aggressive than ant pairs from the same colony.

(d) χ^2 is significant, $p < 0.01$; the counts in the table suggest that ant pairs from the same colony are less likely to be aggressive than ant pairs from different colonies.

(e) χ^2 is significant, $p < 0.01$; the counts in the table suggest that termite pairs from different colonies are less likely to be aggressive than termite pairs from the same colony.

5-4. George and Mary each claimed to have the better method for chocolate cookies. They decided to conduct a study to determine whose cookies were really better. They each baked some cookies using their own method. George asked a random sample of *his* friends to taste his cookies and to complete a questionnaire on their quality. Mary asked a random sample of *her* friends to complete the same questionnaire for her cookies. They then compared the results. Which of the following statements about this study are ***not*** false?

(a) Because George and Mary have a different population of friends, their sampling procedure makes it difficult to compare the recipes.

(b) Because George and Mary each used only their own respective methods, their cooking ability is confounded with the recipe quality.

(c) Because George and Mary each used only the ovens in their houses, the method quality is confounded with the characteristics of the oven.

(d) Because George and Mary used the same questionnaire, their results will generalize to the combined population of their friends.

(e) Because George and Mary each baked one batch, there is no replication of the cookie methods.

6. Consider 18 pairs of numbers $(x1, y1), (x2, y2), \ldots,$ and (x_n, y_n). The mean and standard deviation of the x-values are $x = 5$ and $s_x = 4$, respectively. The mean and standard deviation of the y-values are $y = 10$ and $s_y = 10$, respectively. Moreover, $r_{xy} = 0.2$

Note: x: disposable income per month (NT$ thousand)

y: coffee consumption (cups per week) for university students.

The computer output below

參考用

Predictor	Coefficient	Standard Deviation	
Constant	(a)	2.5	R-Sq = 0.04
x	(b)	0.4	

6-1. (6分) Show the least squares regression line. That is, calculate the values of (a) and (b).

6-2. (3分) Interpret of the value of the slope of the regression line.

6-3. (3分) Interpret the value of R-Sq in the context of this problem

6-4. (6分) Test whether the monthly disposable income is positively affecting weekly consumption of coffee at the 5% statistical level (show your hypotheses and testing process)

7. A sample of 12 students received the following scores on midterm and final examinations in a statistics course

students	1	2	3	4	5	6	7	8	9	10	11	12
Midterm	95	59	88	75	77	73	53	79	88	81	68	62
Final	70	52	88	67	75	85	57	85	60	58	68	80

7-1. (8分) Use $\alpha = 0.05$ and the Wilcoxon signed-rank test to see whether there is significant difference in the two exams?

7-2. (8分) Compute the Spearman rank-correlation for the data and test for a significant correlation with $\alpha = 0.05$

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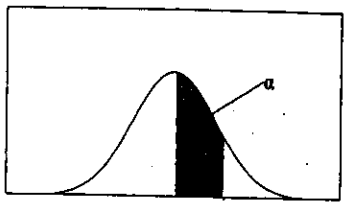
參考用

標準常態累加機率值表

$P(0 < Z < z) = \alpha$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49897	0.49900

Spearman 等級相關係數臨界值表

N	α=.05	α=.025	α=.01	α=.005
5	.900	—	—	—
6	.829	.886	.943	—
7	.714	.786	.893	—
8	.643	.738	.833	.881
9	.600	.683	.783	.833
10	.564	.648	.745	.794
11	.523	.623	.736	.818
12	.497	.591	.703	.780
13	.475	.566	.673	.745
14	.457	.545	.646	.716
15	.441	.525	.623	.689
16	.425	.507	.601	.666
17	.412	.490	.582	.645
18	.399	.476	.564	.625
19	.388	.462	.549	.608
20	.377	.450	.534	.591
21	.368	.438	.521	.576
22	.359	.428	.508	.562
23	.351	.418	.496	.549
24	.343	.409	.485	.537
25	.336	.400	.475	.526
26	.329	.392	.465	.515
27	.323	.385	.456	.505
28	.317	.377	.448	.496
29	.311	.370	.440	.487
30	.305	.364	.432	.478

Wilcoxon 符號等級檢定的臨界值表-成對母體檢定

ONE-TAILED	TWO-TAILED	n=5	n=6	n=7	n=8	n=9	n=10	n=11	n=12
α=.05	α=.10	1	2	4	6	8	11	14	17
α=.025	α=.05		1	2	4	6	8	11	14
α=.01	α=.02			0	2	3	5	7	10
α=.005	α=.01				0	2	3	5	7

		n=13	n=14	n=15	n=16	n=17	n=18	n=19	n=20
α=.05	α=.10	21	26	30	36	41	47	54	60
α=.025	α=.05	17	21	25	30	35	40	46	52
α=.01	α=.02	13	16	20	24	28	33	38	43
α=.005	α=.01	10	13	16	19	23	28	32	37

		n=21	n=22	n=23	n=24	n=25	n=26	n=27	n=28
α=.05	α=.10	68	75	83	92	101	110	120	130
α=.025	α=.05	59	66	73	81	90	98	107	117
α=.01	α=.02	49	56	62	69	77	85	93	102
α=.005	α=.01	43	49	55	61	68	76	84	92

		n=29	n=30	n=31	n=32	n=33	n=34	n=35	n=36
α=.05	α=.10	141	152	163	175	188	201	214	228
α=.025	α=.05	127	137	148	159	171	183	195	208
α=.01	α=.02	111	120	130	141	151	162	174	186
α=.005	α=.01	100	109	118	128	138	149	160	171

		n=37	n=38	n=39	n=40	n=41	n=42	n=43	n=44
α=.05	α=.10	242	256	271	287	303	319	336	353
α=.025	α=.05	222	235	250	264	279	295	311	327
α=.01	α=.02	198	211	224	238	252	267	281	297
α=.005	α=.01	183	195	208	221	234	248	262	277

		n=45	n=46	n=47	n=48	n=49	n=50
α=.05	α=.10	371	389	408	427	446	466
α=.025	α=.05	344	361	379	397	415	434
α=.01	α=.02	313	329	345	362	380	398
α=.005	α=.01	292	307	323	339	356	373

注意：背面有試題

t 分配臨界值表

$P(t > t_\alpha) = \alpha$

d.f.	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	d.f.
1	3.078	6.314	12.706	31.821	63.656	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30