國立中央大學八十四學年度碩士班研究生入學試題卷

所別: 人力資源管理研究所　甲組 科目: 統計學　　共 2 頁 第 1 頁

Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, for example, $Pr(x \geq 2.5)$, where $x \sim \mathcal{N}(0,1)$.

1. (10 points) Suppose $x_1, ..., x_{n1} \overset{iid}{\sim} \mathcal{N}(\mu_1, \sigma_1^2)$ and $y_1, ..., y_{n2} \overset{iid}{\sim} \mathcal{N}(\mu_2, \sigma_2^2)$ are two independent random samples. Suppose also that $n_1 = n_2$. And we know that the sample estimates for these parameters are: $\bar{x}_1, \bar{x}_2, s_1^2, s_2^2$. A statistician suspects that the variance of the first sample is twice the variance of the second sample. So he specifies his null and alternative hypotheses as the following: $H_0 : s_1^2 = 2s_2^2$ and $H_1 : s_1^2 \neq 2s_2^2$. Also, he wants to test if the sample means are the same for both samples, i.e., $H_0 : \bar{x}_1 = \bar{x}_2$ and $H_1 : \bar{x}_1 \neq \bar{x}_2$.

   (a) Are there any problems with his statement?

   (b) How will you test these two hypotheses? Please specify the statistics and their distributions under the null. Remember to specify the degrees of freedom, if any.

2. (a) (11 points) Suppose we have the following information for two independent samples from two distinct normal populations: $\bar{x}_1, \bar{x}_2, s_1^2, s_2^2, n_1, n_2$, where $\bar{x}_i$, s_i^2, and n_i refer to the sample mean, sample variance, and sample size of sample i ($i = 1, 2$), respectively. k is a known constant. Also, let μ_i and σ_i^2 refer to the population mean and variance of sample i, respectively.

   Please complete the following table. Remember to specify the degrees of freedom, if there are any.

		sampling distributions	
Hypothesis	statistics	small sample size	large sample size
$\mu_1 = 0$	(a)	(e)	(i)
$\mu_1 - \mu_2 = 0$	(b)	(f)	(j)
$\sigma_1^2 = k$	(c)	(g)	-
$\sigma_1^2 = \sigma_2^2$	(d)	(h)	-

   Which of the above tests do not require normality assumption?

   (b) (4 points) Suppose, now, that $\hat{p}_i$ refers to the sample proportion of sample i. Given sample size n_1 and n_2, complete the following table.

Hypothesis	statistics	Distribution (large sample size)
$p_1 = 0$	(k)	(m)
$p_1 = p_2$	(l)	(n)

3. (15 points) Use the following information to answer question(a) to (c). A college professor gives a test that has 10 true-false questions. Two students take the test. Student A, who does not know anything about the subject, answers the questions by tossing a coin. The college professor sets up the following hypothesis, where p represents the probability that a student gets an answer right.

   H_0: The students do not know anything ($p = .5$).
   H_1: The students do know the subject ($p > .5$).

   (a) What is the chance of student A getting exactly 6 correct answers when the null hypothesis is true?

   (b) If the professor decides to reject the null hypothesis when the student gets 8 or more correct answers, what is the size of the Type I error? What if he raises the standard to 9 or more correct answers?

   (c) Student B studies one night before the test, so the probability he gets an answer right is $p = .6$. What is the probability that this student can pass the test when the standard for passing is 8 correct answers.

4. (10 points) Suppose in a certain statistics course, 70% of the students who turn in the assignment on time pass the mid-term examination, while 90% of the students who turn in assignment late pass the exam (strange, isn't it?). Suppose also 50% of the students turn in the homework, 50% of whom turn in the homework on time. Only 40% of those who do not turn in the homework pass the midterm. Now, a student who failed in the exam comes to your office asking for changing grade. How would you answer the following questions:

   (a) What is the probability that the student turned in the homework on time? What is the probability that he or she did not turn in the homework?

   (b) Calculate the percentage of the students who turn in the assignment late and pass the exam.

5. (10 points) Given a sample of size n, is the $(1 - \alpha) \times 100\%$ confidence interval for a parameter always the same as the corresponding critical region for that parameter at α significance level? Briefly explain your answer.

6. (10 points) Fit a binomial density with $n = 4$ to the following data. Apply the chi-square test to see whether the binomial assumption is substantiated. Take $\alpha = 0.05$.

$X = x$	0	1	2	3	4
$\#(X = x)$	10	40	60	50	16

7. (10 points) A survey was conducted by sampling 400 persons who were questioned regarding union membership and attitude toward decreased national spending on social welfare programs. The cross-tabulated frequency counts are presented. Take $\alpha = 0.05$.

	Support	Indifferent	Opposed	Total
Union	112	36	28	176
Nonunion	84	68	72	224
Total	196	104	100	400

8. (20 points) Suppose you are asked to estimate the following model:

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + e_t,$$

where $t = 1, ..., 250$. And you get the following results:

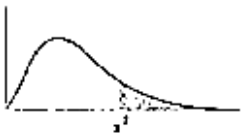
variable	coefficient		std. err.	t-stat	p-value
β_1	-2.474		0.4667	-5.303	0.000
β_2	0.098		0.0069	14.169	0.000
β_3	-0.077		0.0322	-2.389	0.017
β_4	29.286		7.0797	4.137	0.000
R-square	0.69	mean of dependent var.		5.466	
Adjusted R-square	A	S.D. of dependent var.		2.46	
S. E. of regression	B	Sum of squared resid.		C	
F-stat	183.587				

(a) (10 points) What are A, B, and C ?

(b) (10 points) What are the degrees of freedom for the F-statistic? What is the underlying null hypothesis?

Table V　The χ^2 distribution

The first column lists the number of degrees of freedom (ν). The headings of the other columns give probabilities (P) for χ^2 to exceed the entry value. For $\nu > 100$, treat $\sqrt{2\chi^2} - \sqrt{2\nu - 1}$ as a standard normal variable.



ν \ P	0.995	0.975	0.050	0.025	0.010	0.005
1	$0.0^4 3927$	$0.0^3 9821$	3.84146	5.02389	6.63490	7.87944
2	0.010025	0.050636	5.99147	7.37776	9.21034	10.5966
3	0.071721	0.215795	7.81473	9.34840	11.3449	12.8381
4	0.206990	0.484419	9.48773	11.1433	13.2767	14.8602
5	0.411740	0.831211	11.0705	12.8325	15.0863	16.7496
6	0.675727	1.237347	12.5916	14.4494	16.8119	18.5476
7	0.989265	1.68987	14.0671	16.0128	18.4753	20.2777
8	1.344419	2.17973	15.5073	17.5346	20.0902	21.9550
9	1.734926	2.70039	16.9190	19.0228	21.6660	23.5893