國立中央大學95學年度碩士班考試入學試題卷 #_2_頁 第____頁

所別:工業管理研究所碩士班 甲組 科目:微積分

1. A sequence $\{x_n\}$ is said to be a Cauchy sequence if every $\varepsilon > 0$ there exists an integer N such that for all m and n satisfying $m \ge N$ and $n \ge N$, we have $|x_m - x_n| < \varepsilon$.

- (a) (10 points) Show that every convergent sequence is a Cauchy sequence.
- (b) (10 points) Show that every Cauchy sequence is bounded.

2. (15 points) A sequence of functions $\{f_n\}$ converges uniformly on E to a function f if for every $\varepsilon > 0$ there is an integer N such that $n \ge N$ implies $|f_n(x) - f(x)| < \varepsilon$.

Show that if a sequence of functions $\{f_n\}$, defined on E, converges uniformly on E to a function f, then there exists an integer N such that for all m and n satisfying $m \ge N$ and $n \ge N$, $x \in E$ implies $|f_n(x) - f_m(x)| < \varepsilon$.

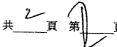
3. (15 points) Let $G = \int_{t=a(x)}^{b(x)} f(t,x) dt$, and let X denote an interval subset of the real

line. Suppose that b(x) > a(x) for all $x \in X$ and both a and b are differentiable on X. Suppose further that f is continuous in the first argument and has a continuous partial derivative for $x \in X$ and $a(x) \le t \le b(x)$. Show

$$\frac{dG}{dx} = \int_{t=a(x)}^{b(x)} \frac{\partial f(t,x)}{\partial x} dt + f(b(x),x)b'(x) - f(a(x),x)a'(x).$$

注:背面有試題

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4. (20 points) The gamma function denoted by $\Gamma(n)$ is defined by

$$\Gamma(n) = \int_{0}^{\infty} x^{n-1} e^{-x} dx$$

which is convergent for n > 0 and $\Gamma(n+1) = n\Gamma(n)$. Evaluate each of the following:

(a)
$$\frac{\Gamma(3)\Gamma(2.5)}{\Gamma(5.5)}$$
, (b) $\int_{0}^{\infty} x^{6}e^{-2x}dx$.

- 5. (10 points) Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$.
- 6. (10 points) Find the maximum and minimum values of $f(x) = 32x 4x^2 12$ subject to the constraint conditions: $2x 30 \le 0$ and $0 \le x \le 6$.
- 7. (10 points) Find the characteristic polynomial, the eigenvalues, and the eigenvectors of $A = \begin{bmatrix} 3 & -4 \\ -5 & 2 \end{bmatrix}$.