1. The series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent. Show that $\sum_{n=1}^{\infty} a_n$ is convergent. (15 points)

2. Suppose f is continuous at a and $f(a) > 0$. Show that there is a number δ such that $f(x) > 0$ for all x satisfying $|x - a| < \delta$. (15 points)

3. Consider a function $f(x)$ with the property that $f^{(3)}$ is continuous on $[a, x]$. Define the remainder term R by
$$f(x) = f(a) + f'(a)(x-a) + R.$$
Show that

$$R(x) = \int_a^x f''(t)(x-t)dt. \text{ (15 points)}$$

4. A point x is a **limit point** of the set A if for every $\varepsilon > 0$ there is a point a in A with $|x - a| < \delta$ but $x \neq a$.

(a) Find all limit points of the following sets. (5 points each)

(i) $\left\{ \dfrac{1}{n} : n \in N \right\}$ 　　　　　　　　　(ii) $\left\{ (-1)^n \left[1 + \dfrac{1}{n} \right] : n \in N \right\}$

(b) Show that x is a limit point of the set A if and only if for every $\varepsilon > 0$ there are infinitely many points a of A satisfying $|x - a| < \delta$. (15 points)

5. Evaluate the following integral
$$\iint_R \frac{\sin(x-y)}{\cos(x+y)} dxdy,$$
where R is the triangular region bounded by the lines $y = 0$, $y = x$, and $x + y = \pi/4$. (15 points)

6. Show that
$$\ln n \leq 1 + \frac{1}{2} + \ldots + \frac{1}{n-1},$$
for $n \in N, n \geq 2$. (Hint: Use the fact that $\ln n = \int_1^n \frac{1}{t} dt$.) (15 points)