

國立中央大學九十學年度碩士班研究生入學試題卷

所別: 工業管理研究所 甲組 科目: 微積分 共 1 頁 第 1 頁

1. (15%) Find (a) $\int_a^b \frac{1}{\sqrt{1+x^2}} dx$,
 (b) $\int_a^b \frac{1}{\sqrt{x^2-1}} dx$, for $a, b > 1$ or $a, b < -1$;
 (c) $\int_a^b \frac{1}{\sqrt{1-x^2}} dx$, for $|a|, |b| < 1$.

Compare your answer for the third integral (c) with that obtained by writing

$$\frac{1}{1-x^2} = \frac{1}{2} \left[\frac{1}{1-x} + \frac{1}{1+x} \right]$$

2. (15%) Decide whether each of the following infinite series is convergent or divergent. The tools that you will need are Leibniz's Theorem and the comparison, ratio, and integral tests.

(a) $\sum_{n=1}^{\infty} \frac{\sin n\theta}{n^2}$ (b) $\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n}$ (c) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$

3. (15%) Find the volume of the solid by rotating the region bounded by the curves $xy=2$, $xy=4$, $x=1$ and $x=2$ about the y -axis.

4. (15%) Find the Taylor series at 0 for the following function: $f(x) = \frac{1}{\sqrt{1-x}} = (1-x)^{-1/2}$

5. (20%) Use the root test to find the radius of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{n}{2^n} z^n$$

6. (20%) Find the following: (a) $\lim_{y \rightarrow 0} \log(1+y)/y$ (b) $\lim_{y \rightarrow \infty} y \log(1+1/y)$