國立中央大學113學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 工業管理組(一般生)　　

科目： 統計學

＊本科考試可使用計算器，廠牌、功能不拘（計算題）

計算題應詳列計算過程，無計算過程者不予計分

1. (a) (10 pts) Show: If $Y \sim Poisson(x)$ and $X \sim \text{Gamma}(k, 1)$ with pdf $f(x) = \frac{x^{k-1}e^{-x}}{\Gamma(k)}$, where k is a positive integer, then

$$\Pr(X > x) = \Pr(Y < k)$$

(b) (15 pts) Suppose that $Z \sim Poisson(\mu)$. Derive the expression for the CDF of Z in terms of the CDF of a Gamma distribution using the result from (a)

2. The joint pdf of random variables X and Y is

$$f(x,y) = \frac{1}{\pi a^2}$$

,where $0 \le x^2 + y^2 \le a^2$ and $a > 0$.

(a) (10 pts) Verify that $f(x,y)$ is a joint density function. Hint: the integration should be one and $\cos^2\theta = (1 + \cos 2\theta)/2$.

(b) (10 pts) Let $X = Rcos(\Theta)$ and $Y = Rsin(\Theta)$. Determine the joint pdf of R and Θ.

(C) (5 pts) Determine the marginal pdf of Θ.

注意：背面有試題

所別： 工業管理研究所 碩士班 工業管理組(一般生)

科目： 統計學

＊本科考試可使用計算器，廠牌、功能不拘

3. In Taiwan, you will get an uniform invoice after making every purchase. Every two months, there will be an uniform invoice lottery, and you can check your collecting uniform-invoices with the winning numbers to see if you win any prizes. Suppose the winning/bingo rate of any uniform invoice is p. Let's use a random variable, N, to represent the number of your collecting uniform-invoices during the two months period. Assume the distribution of N is Poisson with mean 60.
   (a) Suppose the winning/bingo rate of any uniform invoice is $p = 0.005$, what is the probability that none of you collecting uniform invoices during these two months wins any prize? (5 pts)
   (b) Given $p = 0.005$, what is the distribution of the number of winning uniform-invoices? (7 pts)
   (c) Given $p = 0.005$, what is the expected number of winning uniform-invoices. (3 pts)
   (d) Suppose none of your collecting uniform-invoices last November and December won any prize, based on this experience, what is the maximal likelihood estimate of p? (5 pts)
   (e) Suppose during the whole year last year (6 times of lottery), the numbers of your winning uniform-invoices were 0, 1, 1, 0, 0, and 0. Based on these 6 lotteries, what is the maximal likelihood estimate of p? (5 pts)

4. Shewhart control chart is one of the statistical process control schemes, which uses "$\bar{x}$" as an indicator of whether the process is in control or out of control. For example, *temperature* is used to measure the output of a production process. When the process is in control, the mean of the process is $\mu = 200$ and the standard deviation is $\sigma = 0.5$.
   (a) If samples of size 6 are to be used, to monitor the process, please provide the LCL (lower control limit) and UCL (upper control limit) at 5% significance. (6 pts)
   (b) Is the process in control for a sample providing the following data? (2 pts)
   200.9 199.3 199.1 200.5 199.9 201.8
   (c) At the next sample point, the observed data are as following. Is the process still in control? (2 pts)
   200.9 200.5 200.7 199.3 200.1 198.2
   (d) At the next sample point following (c), the observed data are as following. Is the process still in control? (2 pts)
   200.7 200.6 200.5 199.2 200.0 198.1
   (e) Use the observed data in (b) and (c). Please conduct the test and determine whether the mean has changed or not? (4 pts)
   (f) Similarly, please conduct the test and determine whether the mean has changed or not between (c) and (d)? (4 pts)
   (g) Please provide your explanation about the contradiction (if you find any) for the in-control/out-of-control results in (b), (c), (d) and the mean-changed/mean-not-changed results in (e), (f). (5 pts)

注意：背面有試題

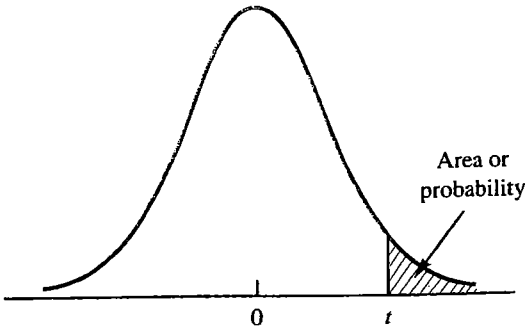
國立中央大學 113 學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 工業管理組(一般生)

科目： 統計學

＊本科考試可使用計算器，廠牌、功能不拘

t DISTRIBUTION



Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail				
	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576