國立中央大學107學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 不分組(一般生)　　共4頁 第1頁

科目： 作業研究

本科考試禁用計算器　　*請在答案卷(卡)內作答

1. (6 * 5 = 30 points)

In a health clinic, the average rate of arrival of patients is 12 patients per hour. On an average, a doctor can serve patients at the rate of one patient every four minutes. Assume, the arrival of patients follows a Poisson distribution and service to patients follows an exponential distribution.

(1) Utilization rate = ___________　　$p = \frac{\lambda}{\mu}$

(2) Average number of patients in the clinic = ___________　　$L = \frac{\lambda}{\mu - \lambda}$

(3) Average number of patients in the waiting line = ___________　　$L_Q = pL$

(4) Average waiting time in the clinic = ___________　　$W = \frac{1}{\mu - \lambda}$

(5) Average waiting time in the waiting line = ___________　　$W_Q = pW$

$P_n = (1 - p)p^n$

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參考用

2. (10 * 2 = 20 points)

Consider a manufacturing firm that receives shipment of parts from two suppliers. Let A_1 denote the event that the parts are received from supplier 1; A_2 is the event the parts are received from supplier 2. They get 65% of the parts from supplier 1 and 35% from supplier 2. In addition, quality levels differ between suppliers (G denote that a part is good and B denote the event that a part is bad).

Suppose a bad part broke one of the machines, what would be the probability the part came from suppler 1? ________________

(Please use below tables and formula to solve.)

	Good Parts (G)	Bad Parts (B)
Supplier 1	98%	2%
Supplier 2	95%	5%

Events A_i	Prior Probabilities $P(A_i)$	Conditional Probabilities $P(B \mid A_i)$	Joint Probabilities $P(A_i \cap B)$	Posterior Probabilities $P(A_i \mid B)$
A_1	0.65			
A_2	0.35			
	1.00		$P(B) = 0.0305$	1.0000

Bayes' Theorem

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \ldots + P(A_n)P(B \mid A_n)}$$

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參考用

3. (15 points) Please solve the following linear programming model with ONE of these methods: the simplex method, the simplex tableau method (簡捷圖表), the dual simplex method, or the dual simplex tableau method. (Note: you will get 0 points if you solve with any other methods.)

$$\textbf{Minimize}\quad 2x_1+15x_2+5x_3+6x_4$$
$$\textbf{Subject to}$$
$$-x_1-6x_2-3x_3-x_4\le -2$$
$$4x_1-10x_2+8x_3-6x_4\ge 6$$
$$x_1,x_2,x_3,x_4\ge 0$$

4. (3 questions; 35 points) This problem is about developing a mixed-integer linear programming (MILP) model which is capable of making optimal decisions on (1) the units of solar panels (太陽能板) (that is, how many pieces of solar panels are needed) and (2) the storage capacity of batteries (電池儲電容量) (that is, how much electricity can be saved in the batteries) so that a convenient store (such as the 7-Eleven or Family Mart nearby your home) can just use solar energy to satisfy all its electricity needs (that is, there is no need for the convenient store to get electricity from Taiwan Power Company (台電)).

The following are four **decision variables** that you can use to develop this MILP model (you are not allowed to define any other decision variables):

- the units of solar panels (denoted by $\boldsymbol{S}$, $\boldsymbol{S}=1,2,3,4,\ldots$),
- the batteries' total storage capacity (denoted by $\boldsymbol{B}$, $\boldsymbol{B}\ge 0$),
- the volume of electricity that is saved in the batteries at the end of day t (denoted by $\boldsymbol{I}_t$ where t represents one of the 365 days of the year, that is, $1\le t\le 365$), and
- the volume of electricity that is wasted (浪費) during day t (denoted by $\boldsymbol{W}_t$, $1\le t\le 365$).

4(a) (10 points) Let $\mathbf{c}^{\text{solar}}$ be the cost for buying each piece of solar panels and $\mathbf{c}^{\text{battery}}$ be the cost for buying each unit of battery storage capacity. The objective for this MILP is to spend a minimum total cost to buy solar panels and batteries which can provide enough electricity to the convenient store. Please write a linear equation (線性方程式) which can be used as the **objective function** of the MILP.

4(b) (10 points) There is an obvious relationship that must be maintained between $\boldsymbol{B}$ and $\boldsymbol{I}_t$: The volume of electricity that can be saved in the batteries at the end of the day can never be more than the batteries' total storage capacity. Please write a linear equation that can be used as a **constraint** to enforce such a relationship.

注意:背面有試題

4(c) (15 points) In addition to the constraint that you have developed in 4(b), you will need another constraint to govern solar energy's generation, consumption and storage for the convenient store. Let $\mathbf{e}_t$ be the volume of solar energy that each piece of solar panel can generate during day t. (This $\mathbf{e}_t$ is a parameter, which means we know its value: In Taiwan, $\mathbf{e}_t$ is about 0.30~0.35 kWh (度) per day; if you are in northern Taiwan, $\mathbf{e}_t$ is smaller; if you are in southern Taiwan such as Pingtung, $\mathbf{e}_t$ is higher.) Furthermore, Let $\mathbf{d}_t$ be the volume of electricity that the convenient store will need during day t. (This $\mathbf{d}_t$ is also a parameter: For an average-sized convenient store, $\mathbf{d}_t$ is about 100~200 kWh per day.)

For day t, the total volume of electricity that solar panels can generate for the convenient store is $\boldsymbol{S} \times \mathbf{e}_t$. Sometimes, $\boldsymbol{S} \times \mathbf{e}_t$ is smaller than $\mathbf{d}_t$. If that happens, the convenient store will have to use the electricity saved (or left) in the batteries at the end of the previous day (that is, $\boldsymbol{I}_{t-1}$) to satisfy all electricity needs. Sometimes, $\boldsymbol{S} \times \mathbf{e}_t$ is larger than $\mathbf{d}_t$. If that happens, there will be extra solar energy that can be saved in the batteries (so that the convenient store can use that electricity in a later day when necessary). However, because the batteries have a fixed storage capacity (that is, $\boldsymbol{B}$), there will be days in which some solar energy will be wasted. (For example, suppose the batteries are already full when the day begins and then solar panels generate a very big volume of electricity during the day due to very good sunlight. In such a situation, $\boldsymbol{S} \times \mathbf{e}_t$ is much larger than $\mathbf{d}_t$ and because the batteries are already full, much of the solar energy will be wasted, that is, the convenient store does not need but cannot save in the batteries, either.)

Please write a linear equation to determine the value of $\boldsymbol{I}_t, 1 \le t \le 365$. (Hint: The value of $\boldsymbol{I}_t$ is affected by $\boldsymbol{I}_{t-1}$, $\boldsymbol{S} \times \mathbf{e}_t$, $\mathbf{d}_t$ and $\boldsymbol{W}_t$.)

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參考用