國立中央大學106學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 不分組(一般生)　　　　共4頁　第1頁

科目： 作業研究

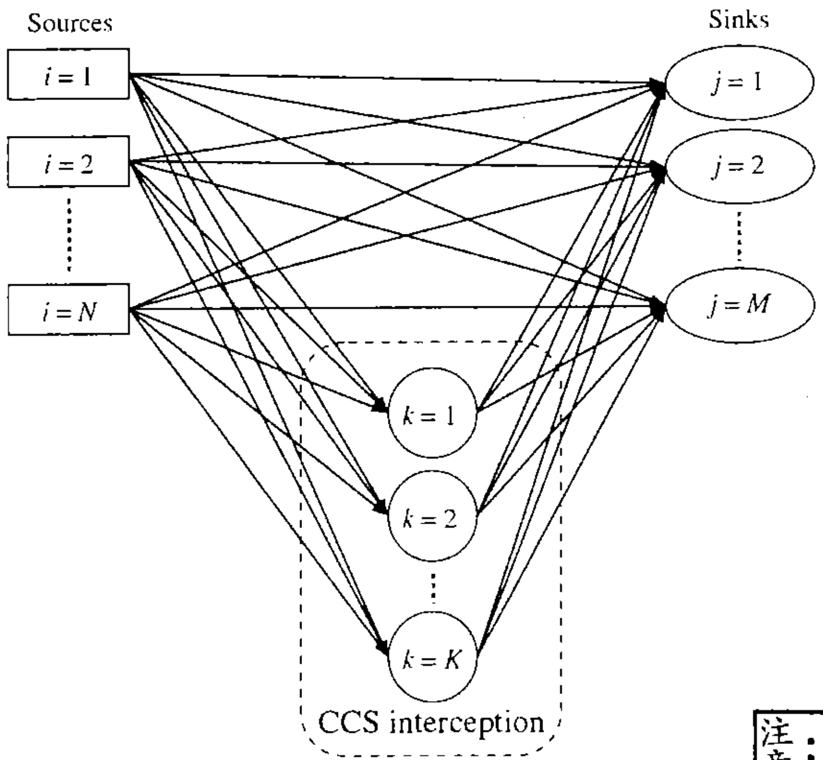
本科考試禁用計算器　　　　*請在答案卷　內作答

須有詳細計算過程

1. (50%) Please propose a general mixed-integer linear programming model for the energy production and allocation problem described in the following; then, find an optimal solution for the numerical data that is given on pages 2~3 using branch-and-bound and simplex tableau methodologies.
[**Note:** You will receive NO points if you solve this problem with different approaches.]

According to *International Energy Agency* (IEA), the worldwide energy demands have been increasing at an average rate of 1.6% per year, and the total required volume will probably be as much as 7.0×10^{20} J per year by 2030. IEA also predicts that 80% of these energy needs will be met by burning fossil fuels, thereby releasing a huge amount of carbon dioxide (CO_2) into the atmosphere and affecting global climate significantly. Therefore, something must be done to mitigate these impacts as much as possible.

Currently, there are "CO_2 capture and storage (CCS)" technologies which can effectively lower the amount of CO_2 released into the atmosphere due to our burning of fossil fuels. To make a very long story short, the use of CCS technologies will require the existing fossil fuel burning process to be modified, and there will also be new equipment to be installed. The following is a figure to conceptually illustrate the use of CCS technologies in an existing energy production and allocation network in which "sources" represent locations where fossil fuels are burned to supply energy and "sinks" represent locations where energies are needed.



參考用

注意：背面有試題

The following defines symbols which you will please use to formulate the problem.

Sets and indices:

I set of energy supply sources, indexed by $i, 1 \le i \le N$

J set of energy demand sinks, indexed by $j, 1 \le j \le M$

For simplicity, we will assume that there is only one CCS technology to be used for all energy sources and sinks, that is, $K = 1$.

Parameters:

S_i maximum amount of energy which source i can produce

α_i^{old} cost incurred when source i produces a unit amount of energy using the conventional approach (i.e., the old way)

e_i^{old} amount of CO_2 which will be released into the atmosphere when source i produces a unit amount of energy using the old way

β_{ij}^{ccs} fixed cost for source i to produce and send energies to sink j using CCS technologies (this cost is needed for source i to supply any amount of energy to sink j using CCS technologies; however, even if CCS is used, note that source i can still send energy to sink j using the old way as long as it is appropriate to do so)

α_i^{ccs} cost incurred when source i produces a unit amount of energy using CCS technologies

e_i^{ccs} amount of CO_2 which will be released into the atmosphere when source i produces a unit amount of energy using CCS technologies

D_j total amount of energy demand at sink j

E_j maximum total amount of CO_2 which is allowed for sink j to receive energy from all sources

(a) (30%) Please propose a mixed-integer linear programming model to decide how much energy to produce and allocate using the above symbols. Your model must have a general formulation so it can apply to any given numerical data. The objective of your model is to minimize the total energy production and allocation costs, while considering the limitation of total energy supply at each source i and energy demand plus the maximum allowable CO_2 emissions at every sink j.

(b) (20%) Use the methodology of branch-and-bound and simplex tableau to identify an optimal solution for the numerical data given on the next page. You must show a detailed calculation process. When you are done, please fill out Table 3 with your optimal solution.

Table 1: Energy demands and emission limits.

sink	Energy demands D_j	Emission limits E_j
1	1,000	600
2	400	300

Table 2: Supply at energy sources with various cost items.

source	S_i	α_i^{old}	e_i^{old}	β_{ij}^{ccs}, $j = 1 \& 2$	α_i^{ccs}	e_i^{ccs}
1 (fossil fuel)	1,400	1	4	1,000	2	1
2 (clean energy)	800	5	0	not applicable	not applicable	not applicable

There are two energy sources in the above table: fossil fuel and clean energy. The old way to use fossil fuels to produce energy has the lowest unit cost ($\alpha_1^{old} = 1$), but it also releases the most CO_2 ($e_1^{old} = 4$). Producing clean energy is the most expensive ($\alpha_2^{old} = 5$) but releases no CO_2 (therefore, it doesn't need CCS to reduce CO_2 emissions anymore). Using CCS on fossil fuels is quite effective since e_1^{ccs} is only 1, but its unit production cost will increase ($\alpha_i^{ccs} = 2$) and also require a fixed cost to be paid ($\beta_{11}^{ccs} = \beta_{12}^{ccs} = 1{,}000$).

Table 3: Your optimal solution (**please make a copy of this table in your answer book and provide an answer to each entry which is marked with "?").

	amount of energy to be allocated		
	fossil fuel		
sink	energy sent using the old way	energy sent using CCS	clean energy
1	?	?	?
2	?	?	?
Your minimum total cost is ___?___.			

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2. (10 * 2 = 20 points)

A toy store sells Barbie dolls. The daily demand is normally distributed with a mean of 50 units and a standard deviation of 6 units, assuming demand occurs in the whole year, i.e., 365 days. The lead-time to receive an order from the manufacturer is constant and equals 5 days. Order cost equals $20, and carrying cost equals $ 0.2 per unit. To achieve a 95% service level, determine

(1) EOQ =

(2) Reorder point =

Note: Fill up the formula with numerical values only, <u>No need to calculate!!!</u>

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c}}$$

$$r = E(D_L) + Z\sqrt{L\sigma_D^2}$$

z = 1.645 for a 95% service level.

3. (5 * 6 = 30 points)

A mechanic is able to install new auto parts at an average rate of 3 per hour, according to a negative exponential distribution. Customers seeking this service arrive at the shop on the average of 2 per hour, following a Poisson distribution. They are served on a first-in, first-out basis and come from a very large population of possible buyers. Please measure the operating performance of the queuing system with below estimates.

A. Number of cars in the system, on average
B. The average waiting time in the system
C. Number of cars waiting in line, on average
D. The average waiting time per car
E. The percentage of time mechanic is busy
F. Probability there are 0 cars in the system

$P_0 = 1 - (\lambda/\mu)$

$P_n = [1 - (\lambda/\mu)](\lambda/\mu)^n$

$L = \lambda/(\mu - \lambda)$

$L_q = \lambda^2/[\mu(\mu - \lambda)]$

$W = 1/(\mu - \lambda)$

$W_q = \lambda/[\mu(\mu - \lambda)]$

$P_w = \lambda/\mu$

$\rho = \lambda/\mu$