1. (10 分) The following is a linear programming model ***P*** and its dual problem ***D***.

$$\begin{array}{llll} \boldsymbol{P}\text{: Minimize} & \mathbf{cx} & \qquad \boldsymbol{D}\text{: Maximize} & \mathbf{wb} \\ \text{Subject to} & \mathbf{Ax} \geq \mathbf{b} & \qquad \text{Subject to} & \mathbf{wA} \leq \mathbf{c} \\ & \mathbf{x} \geq \mathbf{0} & & \mathbf{w} \geq \mathbf{0} \end{array}$$

Suppose we know that ***P*** has an optimal solution; also, we know that the first constraint of ***P*** is redundant (***P*** 的第一條限制式是多餘的，亦即，移除該限制式不會改變問題的最佳解). Now, suppose w_1 is the decision variable in ***D*** that corresponds to the first constraint of ***P***. Then, what is the value of w_1 in an optimal solution to ***D***? Please design a small example and use the example to answer this question.

2. (30 分) Consider a scenario in which you are asked to use operations research to determine the assignment (指派) of a total of A courses to a total of B classrooms in a total of C time periods. Let X_{abc} be a 0/1 (i.e. binary) variable which represents the decision of assigning course a (a could be a Calculus or an English Conversation course) to classroom b (b could be 管理學院 1001 教室) in time period c (c could be 9:00 ~ 9:50 on Monday morning). Then if $X_{abc}=1$, it means course a is assigned to classroom b during time period c, and if $X_{abc}=0$, it means course a is not assigned to classroom b during time period c.

   a. (10 分) Suppose we have this constraint in the model: $\sum_{a=1}^{A} X_{abc}=1$, for any classroom $b = 1, 2, \ldots, B$ and any time period $c = 1, 2, \ldots, C$. What is the purpose of that constraint? (請問該限制式的作用為何？)

   b. (10 分) Please formulate the following requirement as a constraint: "For any course a, a can only be assigned to just one classroom in any time period."

   c. (10 分) Let M be a set which contains every time period on Monday morning. Suppose we need this constraint $\left(\sum_{b=1}^{B}\sum_{c\in M} X_{a_1bc}\right)\left(\sum_{b=1}^{B}\sum_{c\in M} X_{a_2bc}\right)=0$ for assigning two special courses a_1 and a_2 to classrooms. What is the purpose of that constraint?

3. (10 分) Does the following model have an optimal solution? If the answer is yes, please identify <u>all</u> optimal solutions. If the answer is no, please explain why you think the problem does not have an optimal solution.

$$\begin{array}{ll} \text{Minimize} & 6x_1 - 6x_2 + 2x_3 \\ \text{Subject to} & 2x_1 + 6x_2 - 4x_3 \geq 10 \\ & -6x_1 - 4x_2 + 2x_3 \leq 8 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

參考用

注意：背面有試題

4. (5 * 5 = 25 points)

A bank has established two counters one for commercial banking and the second for personal banking. Arrival and service rates at the commercial counter are 6 and 12 per hours respectively. The corresponding numbers at the personal banking counter are 12 and 24 respectively. Assume that arrivals occur in Poisson processes and service times have exponential distributions. Assuming that the two counters operate independently of each other, determine

A. the expected number of waiting customers

B. the mean waiting time at each counter.

	Commercial	Personal
λ		
μ		
$\rho = \frac{\lambda}{\mu}$		
$L_q = \frac{\rho^2}{1-\rho}$		
$W_q = \frac{\rho}{\mu(1-\rho)}$		

(2) What is the effect of operating the two queues as a two server queue with arrival rate 18/hr and service rate 18/hr? Please determine

A. the expected number of waiting customer

B. the mean waiting time at the counter.

C. Is this new system more efficient than the two single-server operations in (1)?

	Two-server queue
λ	
μ	
Number of servers (s)	
$\rho = \frac{\lambda}{s\mu}$	
$\alpha = \frac{\lambda}{\mu}$	
$p_0 = [\sum_0^1 \frac{\alpha^r}{r!} + \frac{\alpha^2}{2(1-\rho)}]^{-1}$	
$L_q = \frac{\rho\alpha^2 p_0}{2(1-\rho)^2}$	
$W_q = \frac{\alpha^2 p_0}{(2)\ 2\mu(1-\rho)^2}$	

注意：背面有試題

5. (10 + 15 = 25 points)

A computer device can be either in a busy mode (state 1) processing a task, or in an idle mode (state 2), when there are no tasks to process. Being in a busy mode, it can finish a task and enter an idle mode any minute with the probability 0.2. Thus, with the probability 0.8 it stays another minute in a busy mode. Being in an idle mode, it receives a new task any minute with the probability 0.1 and enters a busy mode. Thus, it stays another minute in an idle mode with the probability 0.9. The initial state is idle. Let Xn be the state of the device after n minutes.

A. Find the distribution of X_2.

B. Find the steady-state distribution of X_n.