國立中央大學102學年度碩士班考試入學試題卷

所別：工業管理研究所碩士班 不分組(一般生)　科目：作業研究　共 2 頁　第 1 頁

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

參考用

1. (15 分) Suppose the following linear programming (LP) model has a feasible solution.

$$\begin{aligned} &\text{Minimize } \mathbf{cx} \\ &\text{Subject to } \mathbf{Ax} = \mathbf{b} \\ &\qquad\qquad \mathbf{x} \geq \mathbf{0} \end{aligned}$$

In the above, **A** is the coefficient matrix, **x** is the variable vector, **b** is the right-hand-side vector, and **c** is the cost vector. Now consider the following LP:

$$\begin{aligned} &\text{Minimize } \mathbf{1x'} \\ &\text{Subject to } \mathbf{Ax} + \mathbf{x'} = \mathbf{b} \\ &\qquad\qquad \mathbf{x} \geq \mathbf{0}, \mathbf{x'} \geq \mathbf{0} \end{aligned}$$

$\mathbf{1} = [1, 1, ..., 1]$ is a row vector in the above LP. What is the optimal objective function value for the above LP? Please also provide a good reason to explain your answer.

2. (20 分) Let I_{mat} be the inventory of part m at factory a at time t, P_{mat} the production quantity (投入生產的數量) of part m at factory a at time t, $YIELD_{mat}$ the production yield for part m at factory a at time t (生產良率，為單位生產投入量可產出的好產品數量), CT_m the units of time required to produce part m (生產 m 所需要的時間長度), and S_{mat} the quantity of part m shipped from factory a to the customers at time t (出貨量). Assume that when the production of part m is complete, the good parts m are either shipped to the customer or become inventory.

Please complete the following equation, which is a constraint specifying the relationship between I_{mat}, P_{mat}, $YIELD_{mat}$, CT_m, and S_{mat} for a production planning LP.

$$I_{mat} = I_{ma(t-1)} + \text{.....}$$

3. (15 分) Please find an optimal solution to the following LP model.

$$\begin{aligned} &\text{Maximize } 10x_1 + 24x_2 + 20x_3 + 20x_4 + 25x_5 \\ &\text{Subject to } 2x_1 + 2x_2 + 4x_3 + 6x_4 + 10x_5 \leq 38 \\ &\qquad\qquad 2x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 \leq 57 \\ &\qquad\qquad x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

注意：背面有試題

4. (3 * 5 = 15 points)
Customers arrive at a convenience store at the rate of 30 per hour. There is one clerk on duty, who takes an average of 1 minute to serve each customer. Service times are approximately exponential. Calculate
(1) Is this post office in a steady state? Why?
(2) the mean customer time spent receiving or waiting for service
(3) the mean number of customers in line
(4) the mean number of customers receiving or waiting for service
(5) the probability there is no customer in line.

5 (= 20 points)
Consider a society with three social class states. Suppose that intergenerational mobility is characterized by a transition matrix (as below **P**) which does not change over time.

$$P = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.3 & 0.4 & 0.3 \\ 0 & 0.7 & 0.3 \end{bmatrix}$$

What will be the distribution of these three class states in the long run?

Hint: $\boldsymbol{\pi} = \boldsymbol{\pi P}$

6. (15 points)
1% of grownups at age 40 who participate in routine screening have cancer. 80% of them with cancer get positive mammography. 9.6% of them without cancer get positive mammography. A 40-year old man participates in routine screening and has a positive mammography. What's the probability he has cancer?

Hint;

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B|A_1)P(A_1)+ P(B|A_2)P(A_2)+\ldots+ P(B|A_n)P(A_n)}$$