所別：財務金融學系碩士班 丙組 科目：微積分

CALCULUS EXAM

No calaulator is allowed. The probability density function of a standard normal distribution is $\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$, where $\pi \doteq 3.14159$ and $e \doteq 2.71828$. The notation $\mathbf{R}$ is the field of all real numbers. Justifiy all your answers as clearly as possible. Do not use any imaginary number, e.g., $i = \sqrt{-1}$ as a final answer of any questions.

1. Let a_1, a_2, a_3, a_4, a_5 be five positive constants. Find the global maximum of the function $f : \mathbf{R} \to \mathbf{R}$.

   $$f(x)=\begin{cases} (\frac{1}{x})^5 & \text{if } a_i \le x \text{ for } i = 1,2,3,4,5 \\ 1 & \text{otherwise} \end{cases}$$ and the value x that maximizes the function.
   (10%)

2. Find the integer that is closest to the value

   $$\frac{1}{14} \cdot (1.0000002)^{20000000}$$

   Justify your answer. (10%)

3. Let (x, y, z) be a point on the unit sphere $x^2 + y^2 + z^2 = 1$. Find the maximum and the minimum of the function $f(x, y, z) = 20xyz^2$. (10%)

4. Let (x, y, z) be a point in the region encompassed by the six surfaces $x = 3, x = -3, y = 4, y = -4, z = 5, z = -5$ in the space; the boundaries are also included in the region. Find all the points in the region that maximize the function $f(x, y, z) = 20xyz^2$ and all the points in the region that minimize the function $g(x, y, z) = 20|xyz^2|$. (10%)

5. Let $f(x, y) = x^y$. Evaluate the limit

   $$\lim_{h \to 0} \frac{f(3 + 4h, 2 + 2h) - f(3, 2)}{h}.$$

   (10%)

6. Evaluate the integral $\int \int x^2 y^2 dx dy$ over the interior of the circle $x^2 + y^2 = 1$. (10%)

7. Find all continuously differentiable functions $f : \mathbf{R} \to \mathbf{R}$ that satisfy

   $$f(x) + 4 f''(x) = x^2.$$

   (10%)

8. Let $A = \int_{-\infty}^{\infty} |e^{-x^2} \sin x| dx$.

   (a) Prove that $A < 2$. (15%)

   (b) Can you write down the integral in a closed form? If so, write down the closed form. If not, write down the procedure to evaluate the integral numerically. (15%)