一、(50 points)

Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, say, $Pr(x \geq 2.5)$, where $x \sim \mathcal{N}(0,1)$.

1. (15 points) Suppose a box has 5 balls labeled $1, 2, ..., 5$. Two balls are drawn from the box without replacement. Let x be the number of the first ball, and y the number of the second ball. Answer the following questions:

   (a) (5 points) Calculate the mean and variance of y.

   (b) (10 points) Calculate the correlation coefficient of x and y.

2. (10 points) Suppose a box has 5 balls labeled $1, 2, ..., 5$. A sequence of balls are independently drawn from the box with replacement. For each of the outcomes, you win \$1 dollar if the number of the ball is greater than or euqal to 3, and lose \$1 if the number is 1 or 2. Suppose you have \$10 dollars before the games start. Answer the following questions:

   (a) (5 points) What is your expected "wealth" after the 10 games.

   (b) (5 points) What is the probability that you will lose half of your money after the 10 games.

3. (25 points) Let x and y have the following joint probability density function (pdf):

$$f(x,y) = \begin{cases} 1 & -x < y < x \\ 0 & \text{otherwise,} \end{cases}$$

   where $0 < x < 1$. Answer the following three questions:

   (a) (10 points) Suppose the value of x is known, calculate the conditional mean and variance of y, i.e., $E(y|x)$ and $var(y|x)$.

   (b) (10 points) Calculate the unconditional mean and variance of y, i.e., $E(y)$ and $var(y)$.

   (c) (5 points) Calculate the probability that $y \geq \frac{1}{2}$, i.e., $Pr(y \geq \frac{1}{2})$.

參考用

二、(50 points)

1. Supporters claim that a new windmill can generate an average of at least 800 kilowatts of power per day. Daily power generation for the windmill is assumed to be normally distributed with a standard deviation of 120 kilowatts. A random sample of 100 days is taken to test this claim against the alternative hypothesis that the true mean is less than 800 kilowatts. The claim will be accepted if the sample mean is 776 kilowatts or more and rejected otherwise.

   (a) What is the probability α of a Type I error using the decision rule if the population mean is in fact 800 kilowatts per day? (5%)

   (b) What is the probability β of a Type II error using this decision rule if the population mean is in fact 740 kilowatts per day? (5%)

   (c) Suppose that the same decision rule is used, but with a sample of 200 days rather than 100 days. Would the vale of α be larger than, smaller than, or the same as that found in (a). (5%)

   (d) Suppose that a sample of 100 observations was taken but that the decision rule was changed so that the claim would be accepted if the sample mean was at least 765 kilowatts. Would the value of β be larger than, smaller than, or the same as that found in (b). (5%)

注意：背面有

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2. The number of customers arriving at a supermarket checkout counter over a period of 200 minutes was recorded, yielding the results shown in the table. The average number of customers per minute was 2.3. Test the null hypothesis that the population distribution is Poisson. (10%)

Number of Customers in Minute:	0	1	2	3	4	5 or more
Observed Frequency :	16	50	51	44	28	11

3. Students were classified according to three parental income groups and also according to three possible score ranges in the SAT examination. One student was chosen randomly from each of the nine cross-classifications, and the grade point average of each sample member at the end of the sophomore year was recorded. The results are shown in the accompanying table.

SAT Score	Income Group		
	High	Moderate	Low
Very high	3.7	3.6	3.6
High	3.4	3.5	3.2
Moderate	2.9	2.8	3.0

(a) Set out the analysis of variance table. (10%)

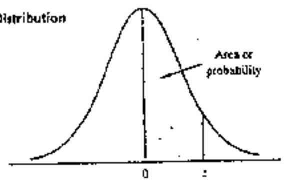
(b) For the two-way analysis of variance model with one observation per cell, write the observation from the *i*th group and *j*th block as

$X_{ij} = \mu + G_i + B_j + \varepsilon_{ij}$

Consider the observation on moderate income group and high SAT score (X_{22}=3.5). Estimate and interpret G_2 and B_2. (10%)

參考用

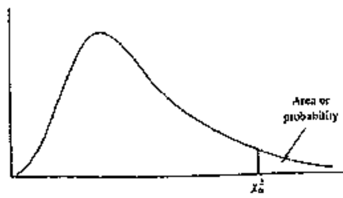
Standard Normal Distribution



Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for z = 1.25 the area under the curve between the mean and z is .3944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Chi-Square Distribution



Entries in the table give χ^2_α values, where α is the area or probability in the upper tail of the chi-square distribution. For example, with 10 degrees of freedom and a .01 area in the upper tail, $\chi^2_{.01}$ = 23.2093.

Degrees of Freedom	Area in Upper Tail .995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	392,704 × 10^{-10}	157,088 × 10^{-9}	982,069 × 10^{-9}	393,214 × 10^{-8}	.0157908	2.70554	3.84146	5.02389	6.63490	7.87944
2	.0100251	.0201007	.0506356	.102587	.210720	4.60517	5.99147	7.37776	9.21034	10.5966
3	.0717212	.114832	.215795	.351846	.584375	6.25139	7.81473	9.34840	11.3449	12.8381
4	.206990	.297110	.484419	.710721	1.063623	7.77944	9.48773	11.1433	13.2767	14.8602
5	.411740	.554300	.831211	1.145476	1.61031	9.23635	11.0705	12.8325	15.0863	16.7496
6	.675727	.872085	1.237347	1.63539	2.20413	10.6446	12.5916	14.4494	16.8119	18.5476
7	.989265	1.239043	1.68987	2.16735	2.83311	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.344419	1.646482	2.17973	2.73264	3.48954	13.3616	15.5073	17.5346	20.0902	21.9550
9	1.734926	2.087912	2.70039	3.32511	4.16816	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.15585	2.55821	3.24697	3.94030	4.86518	15.9871	18.3070	20.4831	23.2093	25.1882
11	2.60321	3.05347	3.81575	4.57481	5.57779	17.2750	19.6751	21.9200	24.7250	26.7569
12	3.07382	3.57056	4.40379	5.22603	6.30380	18.5494	21.0261	23.3367	26.2170	28.2995
13	3.56503	4.10691	5.00874	5.89186	7.04150	19.8119	22.3621	24.7356	27.6883	29.8194
14	4.07468	4.66043	5.62872	6.57063	7.78953	21.0642	23.6848	26.1190	29.1413	31.3193
15	4.60094	5.22935	6.26214	7.26094	8.54675	22.3072	24.9958	27.4884	30.5779	32.8013
16	5.14224	5.81221	6.90766	7.96164	9.31223	23.5418	26.2962	28.8454	31.9999	34.2672
17	5.69724	6.40776	7.56418	8.67176	10.0852	24.7690	27.5871	30.1910	33.4087	35.7185
18	6.26481	7.01491	8.23075	9.39046	10.8649	25.9894	28.8693	31.5264	34.8053	37.1564
19	6.84398	7.63273	8.90655	10.1170	11.6509	27.2036	30.1435	32.8523	36.1908	38.5822
20	7.43386	8.26040	9.59083	10.8508	12.4426	28.4120	31.4104	34.1696	37.5662	39.9968
21	8.03366	8.89720	10.28293	11.5913	13.2396	29.6151	32.6705	35.4789	38.9321	41.4010
22	8.64272	9.54249	10.9823	12.3380	14.0415	30.8133	33.9244	36.7807	40.2894	42.7958
23	9.26042	10.19567	11.6885	13.0905	14.8479	32.0069	35.1725	38.0757	41.6384	44.1813
24	9.88623	10.8564	12.4011	13.8484	15.6587	33.1963	36.4151	39.3641	42.9798	45.5585
25	10.5197	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141	46.9278
26	11.1603	12.1981	13.8439	15.3791	17.2919	35.5631	38.8852	41.9232	45.6417	48.2899
27	11.8076	12.8786	14.5733	16.1513	18.1138	36.7412	40.1133	43.1944	46.9630	49.6449
28	12.4613	13.5648	15.3079	16.9279	18.9392	37.9159	41.3372	44.4607	48.2782	50.9933
29	13.1211	14.2565	16.0471	17.7083	19.7677	39.0875	42.5569	45.7222	49.5879	52.3356
30	13.7867	14.9535	16.7908	18.4926	20.5992	40.2560	43.7729	46.9792	50.8922	53.6720
40	20.7065	22.1643	24.4331	26.5093	29.0505	51.8050	55.7585	59.3417	63.6907	66.7659
50	27.9907	29.7067	32.3574	34.7642	37.6886	63.1671	67.5048	71.4202	76.1539	79.4900
60	35.5346	37.4848	40.4817	43.1879	46.4589	74.3970	79.0819	83.2976	88.3794	91.9517
70	43.2752	45.4418	48.7576	51.7393	55.3290	85.5271	90.5312	95.0231	100.425	104.215
80	51.1720	53.5400	57.1532	60.3915	64.2778	96.5782	101.879	106.629	112.329	116.321
90	59.1963	61.7541	65.6466	69.1260	73.2912	107.565	113.145	118.136	124.116	128.299
100	67.3276	70.0648	74.2219	77.9295	82.3581	118.498	124.342	129.561	135.807	140.169