國立中央大學八十五學年度碩士班研究生入學試題卷

所別: 財務管理研究所 甲乙組 科目: 統計學 共 2 頁 第 1 頁

Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, say, $Pr(z \geq 2.5)$, where $z \sim N(0,1)$.

PART I

1. (15 points) Suppose $x_1, ..., x_n \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$ and $y_1, ..., y_n \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$ are two independent random samples. Suppose also that the sample estimates for these parameters are: $\bar{x}, \bar{y}, s_x^2, s_y^2$. A statistician suspects that the variance of the first sample is k times the variance of the second sample, where k is a known constant. So he specifies her null and alternative hypotheses: $H_0 : s_x^2 = ks_y^2$ and $H_1 : s_x^2 \neq ks_y^2$. Also, he wants to test if the sample mean of x is twice the sample mean of y, i.e., $H_0 : \bar{x} = 2\bar{y}$ and $H_1 : \bar{x} \neq 2\bar{y}$.

   (a) Are there any problems with his statement?

   (b) How will you test these hypotheses? Please specify the statistics and their distributions under the null. Remember to specify the degrees of freedom, if any.

   (c) Suppose a random variable z is known to have a χ^2 distribution with ν degrees of freedom, and $w = 2z$. Calculate $E(w)$ and $Var(w)$. What do you know about the distribution of w?

2. (15 points) Use the following information to answer questions (a) - (c). A college professor gives a test that has 10 true-false questions. Two students take the test. Student A, who does not know anything about the subject, answers the questions by tossing a coin. The college professor sets up the following hypothesis, where p represents the probability that a student gets an answer right.

   H_0: The students do not know anything ($p = .5$).
   H_1: The students do know the subject ($p > .5$).

   (a) What is the chance of student A getting exactly 6 correct answers when the null hypothesis is true?

   (b) If the professor decides to reject the null hypothesis when the student gets 8 or more correct answers, what is the size of the Type I error? What if he raises the standard to 9 or more correct answers?

   (c) Student B studies one night before the test, so the probability he gets an answer right is $p = .6$. What is the probability that this student can pass the test when the standard for passing is 8 correct answers.

3. (20 points) Suppose you are asked to estimate the following model:

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \epsilon_t,$$

   where $t = 1, ..., 50$. And you get the following results:

parameter	coefficient	std. err.	t-stat	p-value
β_1	-9.790	0.8173	-11.978	0.000
β_2	0.518	0.0479	11.559	0.000
β_3	1.248	0.0987	12.636	0.000
R-square	0.8444	sample mean of y_t	-0.2630	
adjusted R-square	A	standard deviation of y_t	0.2390	
standard error of regression	B	sum of squared residuals	C	
F-statatic	127.4938			

   (a) (10 points) What are A, B, and C ?

   (b) (10 points) The F-statistic given in the table is provided to test "if the model is significant." What are the degrees of freedom for the F-statistic? What is the underlying null hypothesis for this test?

Part II

1. Table below contains the sales data for the soft drink. Each flavor was assigned to five localities for test marketing, and the number of cases sold per 1,000 population during the study period was recorded for each locality.

   (a). Please test whether or not color of the drink affects mean sales. (7 points)
   (b). Explain a 90 percent confidence interval that mean sales for the colorless version. (8 points)
   (c). Using confidence interval to compare the difference effects of green and pink colors. (10 points)

observation	Treatment			
	1 Colorless	2 Pink	3 Orange	4 Green
1	26.5	31.2	27.9	30.8
2	28.7	28.3	25.1	29.6
3	25.1	30.8	28.5	32.4
4	29.1	27.9	24.2	31.7
5	27.2	29.6	26.5	32.8

(F(0.95,2,15)=3.68 F(0.95;3,15)=3.29 F(0.95;2,16)=3.63 F(0.95;3,16)=3.24
F(0.95;4,15)=3.06 F(0.95;4,16)=3.01 t(0.95;15)=1.753 t(0.95;16)=1.746)

2. A business researcher studied the impact of information on credit decisions. Forty volunteer credit managers were grouped into 20 pairs, each pair having similar backgrounds with respect to business experience, education, and so on. Two case descriptions were prepared of a hypothetical business that is requesting trade credit. The first one provided limited information about the business. The second case description provided detailed information, including financial and operating data for the past several years. In each pair, a random number was used to decide which manager received description 1 and which one received description 2. Each manager then studied the description and determined the maximum amount of credit he or she would extend to the business.

The sample results are presented as below:

   (a). please estimate the median of the population of differences with a 95 percent confidence interval. (7 points)
   (b). Please test whether or not the median difference in credit extended with limited and detailed information is zero. (8 points)
   (c). Assuming that the population of difference is symmetrical, how to use more powerful test to validate the difference. (10 points)

Pair of Managers		1	2	3	4	5	6	7	8	9	10
Credit Amount ($000)	Limited Information	85	50	85	70	30	75	67	60	60	85
	Detailed Information	90	60	113	100	45	99	86	85	85	87

Pair of Managers		11	12	13	14	15	16	17	18	19	20
Credit Amount ($000)	Limited Information	70	85	70	77	70	50	55	75	81	20
	Detailed Information	50	90	93	115	62	85	72	125	103	60

(Z(0.95)=1.645 Z(0.975)=1.96)