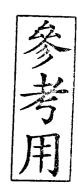
所別:財務金融學系碩士班 甲組(一般生) 科目:統計 共 3 頁 第 1 頁

財務金融學系碩士班 乙組(一般生)

本科考試禁用計算器

\*請在試卷答案卷(卡)內作答



## Answering Problems

State with your reasoning or proofs. Please be precise and concise. No point will be graded if no explanation is provided. (答題請精準、簡捷,並皆須提示理由解釋或證明,否則不予計分。)

- 1. If for an unknown random variable X with  $\mathbb{E}[X]=35$ , we also know that  $P(X \ge 45)=0.055$  and  $P(X \le 25)=0.015$ . Find the lower bound of  $\mathrm{var}(X)$ . (8%)
- 2. Suppose  $\{X_1, X_2, \dots, X_n\}$  is a random sample drawn from a Poisson distribution with parameter  $\lambda$ ,
  - (a) Find the maximum likelihood estimator (MLE) for  $\lambda$ . (4%)
  - (b) Please examine the consistency and sufficiency of the MLE. (8%)
- 3. Suppose the joint distribution of random variables X and Y is expressed as

$$f(x,y) = \begin{cases} \exp(-x), & 0 < y < x < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Please find P(Y > 5|X < 10). (結果以指數表達即可, 5%)
- (b) Find  $\mathbb{E}[X+Y]$  .(5%)
- 4. Given a set of random sampled data, {45, 11, 63, 59, 17, 4, 23, 28, 65, 42}, from population, please find an approximately 95% confidence interval for the population median. (10%)
- 5. Let X and Y be two random variables, prove that

$$var(Y) = \mathbb{E}[var(Y|X)] + var(\mathbb{E}[Y|X]). \tag{10\%}$$

注:背面有試題

## 國立中央大學102學年度碩士班考試入學試題卷

所別:財務金融學系碩士班 甲組(一般生) 科目:統計 共 3 頁 第 2 頁

財務金融學系碩士班 乙組(一般生)

本科考試禁用計算器

\*請在試卷答案卷(卡)內作答



6. To test if t-test statistic is appropriate to use for a given population, we conduct an investigation with the following procedure. We randomly draw 1,000 *samples* from this population. Each of these samples has *n* observations drawn without replacement, and the t-statistic for each sample is calculated as

$$t_k = \frac{\bar{x}_k}{\sigma(x_k)/\sqrt{n}}, k = 1,...,1000,$$

where  $\{x_k\}$  are the observations in sample k, and 1,000 t-statistics  $(t_1, ..., t_{1000})$  are derived. We then compare each of  $t_k$  to the critical values of t-test statistic associated with the two-tailed 5% significance level. Suppose 1,000 samples of size n are sufficient to make a correct inference, and we find that 92 of these 1,000 t-statistics are outside the 95% confidence interval. Based on such a finding, if we use t-statistic to test hypothesis on the observations drawn from this population, is it type I or type II error that we are likely to commit, and why? (6%)

7. Suppose we stand at time t=0 and consider the following model for the time-series dynamics of  $Y_t$ , t=1,2:

$$Y_1 = \beta Y_0 + u_1,$$
  
$$Y_2 = \beta Y_1 + u_2,$$

where the subscript represents time. Residual terms  $u_t$  satisfy

E(
$$u_t$$
)=0 for  $t$ =1,2,  
E( $u_t$ )=  $\sigma^2$  for  $t$ =1,2,  
E( $u_1u_2$ )=  $\sigma_{12}\neq 0$ .

 $Y_0$  is a known number at time t=0. Find  $E(Y_2)$  and  $Var(Y_2)$ . (6%)

- 8. Suppose we need to estimate a linear regression model  $Y_i = \beta X_i + \varepsilon_i$ , where residual terms  $\varepsilon_i$  is independent and identically distributed,  $E(\varepsilon_i) = 0$ ,  $Var(\varepsilon_i) = \sigma^2$ ,  $Cov(\varepsilon_i, \varepsilon_j) = 0$  if  $i \neq j$ , i = 1, ..., N.
  - (a) Find the least square estimate of  $\beta$  (denoted as  $\hat{\beta}$ ). (Note: Be sure to explain why your estimator gives the minimal sum of squared errors) (6%)
  - (b) Find  $E(\hat{\beta})$  and  $Var(\hat{\beta})$ . Is  $\hat{\beta}$  unbiased? (5%)
  - (c) Suppose now we have another estimator  $\widetilde{\beta} = \frac{\sum\limits_{i=1}^{N} Y_i}{\sum\limits_{i=1}^{N} X_i}$ . Is  $\widetilde{\beta}$  unbiased? Which of  $\widetilde{\beta}$  and

 $\hat{\beta}$  is considered a better estimator and why? (6%)

(d) Suppose now  $Var(\varepsilon_i) = \sigma_i^2$ , where  $\sigma_i^2 \neq \sigma_j^2$  if  $i \neq j$ . Is  $\hat{\beta}$  unbiased? What is  $Var(\hat{\beta})$ ? (5%)

## 國立中央大學102學年度碩士班考試入學試題卷

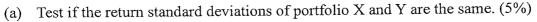
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9. We need to decide whether the returns of two different portfolios follow the same underlying Normal distribution. Suppose  $\{x_1, ..., x_n\}$  and  $\{y_1, ..., y_m\}$  are the historical returns of portfolios X and Y, respectively. Please conduct the following tests with  $\alpha\%$  significance level. (Be sure to clearly specify the tested hypothesis, the used statistics, and the confidence interval.)



- (b) Suppose we conclude the return standard deviations for two portfolios are the same from (a). Test if the means of returns of portfolio X and Y are the same. (5%)
- 10. Consider the following simultaneous equation system describing the mutual influence between the equilibrium price (p) and the quantity (q) of a product over time:

$$p_t = \beta_1 q_t + \beta_2 x_t + u_t, \quad (1)$$

$$q_t = \gamma p_t + v_t, \qquad (2)$$

where subscript t represents time,  $x_t$  is an exogenous variable that affects price, and  $u_t$  and  $v_t$  are residual terms satisfying

$$\mathbf{E}(u_t) = \mathbf{E}(v_t) = 0$$

$$E(u_t^2) = \sigma_u^2, E(v_t^2) = \sigma_v^2$$

$$E(u_t v_t) = E(u_t x_t) = E(v_t x_t) = 0.$$

Show  $Cov(q, u) \neq 0$  and  $Cov(p, v) \neq 0$ . (6%)



