所別：產業經濟研究所碩士班 甲組 科目：統計學

A. 選擇題：(共七題，每題 10 分，請依照題目順序，將正確選項寫在答案卷，違者扣總分 20 分。)

一、(10%) **Case 1:** Consider an experiment in which five fibers having different lengths are subject to learn which fiber will break first. Suppose that the lengths of the five fibers are 1 inch, 2 inches, 3 inches, 4 inches, and 5 inches, respectively. Suppose that the probability that any given fiber will be the first to break is proportional to the length of that fiber.
Case 2: Suppose that three fair coins are tossed simultaneously.

Question:
a) The probability that the length of the fiber that breaks first is not more than 3 inches is 0.3.
b) The probability that the length of the fiber that breaks first is not more than 3 inches is 0.4.
c) The probability of obtaining exactly two heads is 1/8.
d) The probability of obtaining exactly two heads is 3/8.
e) Both (a) and (c) are correct.
f) Both (b) and (d) are correct.
g) Both (b) and (c) are correct
h) None of the above is correct.

二、(10%) **Case 1:** Suppose that the p.d.f. of a certain random variable X has the following form: $f(x)=\begin{cases} cx & for \quad 0<x<4 \\ 0 & otherwise, \end{cases}$
where c is a given constant. **Case 2:** Suppose that the joint p.d.f. of X and Y is specified as $f(x,y)=\begin{cases} cx^2y & for \quad x^2 \le y \le 1 \\ 0 & otherwise, \end{cases}$

Question:
a) The value of c in the case 1 is 1/8.
b) The value of c in the case 2 is 4/21.
c) $\Pr(1 \le X \le 2)$ in the case 1 is 5/16.
d) $\Pr(X \ge Y)$ in the case 2 is 5/20.
e) Both (a) and (b) are correct.
f) Both (a) and (c) are correct.
g) Both (b) and (d) are correct.
h) (a), (b), (c), and (d) are correct.

三、(10%) **Case 1:**Suppose that a random variable X can take each of the five values –2, 0, 1, 3, and 4 with equal probability.
Case 2: Suppose that in a box containing red balls and blue balls, the proportion of red balls is p. Suppose again that a random sample of n balls is selected from the box with replacement. Let $X_i = 1$ if the i th ball that is selected is red, and let $X_i = 0$ otherwise where X denotes the total number of red balls in the sample.

Question:
a) The variance of X in the case 1 is 3.56.
b) The variance of $Y = 4X - 7$ in the case 1 is 72.96.
c) The variance of X in the case 2 is np^2.
d) The moment generating function of X_i in the case 2 is $\psi_i(t) = pe^t$.
e) Both (a) and (b) are correct.
f) Both (a) and (c) are correct.
g) Both (b) and (d) are correct.
h) (a), (b), (c), and (d) are correct.

四、(10%) **Case 1:**Suppose that a random sample is to be taken from a distribution for which the value of the mean μ is unknown, but for which it is known that the standard deviation σ is 2 units. **Case 2:** Suppose that a fair coin is to be tossed n times independently. For $i = 1,....,n$, let $X_i = 1$ if a head is obtained on the i th toss and let $X_i = 0$ if a tail is obtained on the i th toss.

Question:
a) The sample size must be in order to make the probability at least 0.99 that $|\overline{X}_n - \mu|$ will be less than 1 unit in the case 1 is 200.

注：背面有試題

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b) The sample size must be in order to make the probability at least 0.99 that $|\bar{X}_n - \mu|$ will be less than 1 unit in the case 1 is 400.
c) The number of times the coin must be tossed in order to make $\Pr(0.4 \le \bar{X}_n \le 0.6) \ge 0.7$ in the case 2 is 104.
d) The number of times the coin must be tossed in order to make $\Pr(0.4 \le \bar{X}_n \le 0.6) \ge 0.7$ in the case 2 is 84.
e) Both (a) and (c) are correct.
f) Both (b) and (d) are correct.
g) Both (b) and (c) are correct.
h) None of the above is correct.

五、(10%) **Case 1:** Suppose a random sample of 25 observations is taken from a normal distribution with a mean μ and standard deviation 2. What is the probability that the sample mean will lie within one unit of μ ?

Case 2: Suppose that on a certain English examination, students from university A achieve scores which are normally distributed with a mean of 625 and a variance of 100, and that students from university B achieve score which are normally distributed with a mean of 600 and a variance of 150. If two students from university A and three students from university B take this examination, what is the probability that the average of the scores of the two students from university A will be greater than the average of the scores of the three students from university B? (Note Normal distribution function $\Phi(2.0)$=0.9773 $\Phi(2.3)$=0.9893 $\Phi(2.5)$=0.9938 $\Phi(2.8)$=0.9974 $\Phi(3.0)$=0.9987)

The correct answer is:
a) 0.9876 in the case 1.
b) 0.9773 in the case 1.
c) 0.9938 in the case 2.
d) 0.9987 in the case 2.
e) Both (a) and (c) are correct.
f) Both (b) and (d) are correct.
g) Both (b) and (c) are correct.
h) None of the above is correct.

六、(10%) Suppose that 6 observations $X_1,...,X_6$ are selected at random from a normal distribution for which both the mean μ_1 and the variance σ_1^2 are unknown; and that is found that $\sum_{i=1}^{6}(X_i - \bar{X}_6)^2 = 30$. Suppose also that 21 observations $Y_1,...,Y_{21}$ are selected at random from another normal distribution for which both the mean μ_2 and the variance σ_2^2 are unknown; and that is found that $\sum_{i=1}^{21}(Y_i - \bar{Y}_{21})^2 = 40$.

Question:

a) To test the hypothesis that $\mu_1 \le \mu_2$ against $\mu_1 > \mu_2$, the statistic U can be defined as: $U = \dfrac{5(\bar{X}_6 - \bar{Y}_{21})}{\sqrt{70((1/6)+(1/21))}}$
b) To test the hypothesis that $\mu_1 \le \mu_2$ against $\mu_1 > \mu_2$, use the t distribution with 25 degree of freedom.
c) To test the hypothesis that $\sigma_1^2 \le \sigma_2^2$ against $\sigma_1^2 > \sigma_2^2$, the value of the statistic is equal to 3.
d) To test the hypothesis that $\sigma_1^2 \le \sigma_2^2$ against $\sigma_1^2 > \sigma_2^2$, use the F distribution with 5 and 20 degrees of freedom.
e) Both (a) and (b) are correct.
f) Both (a) and (c) are correct.
g) Both (b) and (d) are correct.
h) (a), (b), (c), and (d) are correct.

七、(10%) Suppose that the proportion p of defective items in a large population of manufactured items is unknown and test the hypothesis $p = 0.1$ against $p \ne 0.1$. Suppose also that in a random sample of 100 items, it is found that 16 are defective.

Question:

a) The value of statistic is 3.
b) The degree of freedom of the χ^2 distribution is 2.
c) The value of statistic is 4.
d) The degree of freedom of the χ^2 distribution is 1.
e) Both (a) and (b) are correct.
f) Both (c) and (c) are correct.
g) None of the above is correct.

B. 證明與求解題 (共三題，每題 10 分，請依照題目順序，將正確選項寫在答案卷，違者扣總分 10 分。)

一、(10%) Show that the maximum likelihood estimator of $\hat{\sigma}^2 = \dfrac{1}{n}\sum_{i=1}^{n}(y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)^2$.

二、(10%) Show that $E(\hat{\beta}_2) = \beta_2$.

三、(10%) Find the $Var(\hat{\beta}_1)$.