以下每題 25 分,可選擇以英文或中文作答。

1. Show that continuity is a necessary, but not a sufficient condition for differentiability.

2. Show that under the constraint $u - 2v = 0$, the quadratic form $q = 4u^2 + 4uv + 3v^2$ is always positive.

3. Find the general solution for the difference equation $y_{t+2} + \frac{1}{4}y_t = 5$.

4. Let B be a closed convex set of points in n-dimensional euclidean space, and let $x = (x_1, \cdots, x_n)$ be a point not in B. Show that there exist numbers $p_1, \cdots, p_n, p_{n+1}$ such that $\sum_{i=1}^{n} p_i x_i = p_{n+1}$ and $\sum_{i=1}^{n} p_i y_i > p_{n+1}$ for all $y \in B$.