

國立中央大學八十五學年度碩士班研究生入學試題卷

所別: 產業經濟研究所 組別: 甲組 科目: 基礎數學 共 1 頁 第 1 頁

1. (a). If $y = (x^2 + 1) \cos x$, find $\frac{dy}{dx}$.

(b). Find $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{x}{\ln x} \right)$

(c). Find $\int \frac{x \sin \sqrt{x^2+4}}{\sqrt{x^2+4}} dx$ (50%)

(d). Show that $\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi}$.

(e). Evaluate $\int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2}} dx$

2. Find the minimum distance between the origin and the surface $z^2 = x^2 y + 4$. (10%)

3. Call a function $f(x,y)$ homogeneous of degree n if $f(tx,ty) = t^n f(x,y)$ for all $t > 0$. Show that a differentiable function $f(x,y)$ is a homogeneous of degree n if and only if $f(x,y)$ satisfies (10%)

$$n f(x,y) = x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$$

4. Determine whether

$$f_1(x) = 1, f_2(x) = e^x, \text{ and } f_3(x) = e^{2x} \quad (10\%)$$

form a linearly dependent set or a linearly independent set.

5. Solve the differential equations

$$2(xy + x) + (x^2 + 1) \frac{dy}{dx} = 0 \quad (10\%)$$

6. Let $\begin{pmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{pmatrix}$ (10%)

be the augmented matrix for a linear system. For what values of a and b does the system have

- (a). a unique solution
- (b). a one-parameter solution
- (c). a two-parameter solution
- (d). no solution