國立中央大學八十五學年度碩士班研究生入學試題卷

所別：產業經濟研究所　乙組　科目：乙統計學　共 2 頁 第 1 頁

1.(12%) Let a ball be taken at random from a bowl that contains 15 white balls and 5 red balls.

(1) If it is drawn with replacment, please answer the following questions:

(3%) a. What is the probability that the fourth white ball is drawn in the sixth random experiment?

(3%) b. Define a random variable X as the number of experiments executed when the first white ball is drawn. What distribution is X followed? What is the expected number of experiments to have a white ball drawn?

(2) If it is drawn without replacement, please answer the following questions:

(3%) a. What is the probability that the first white ball is drawn in the sixth radom experiment?

(3%) b. Let Y be the number of white balls drawn in five random experiments. What distribution is Y followed? What is the expected number of white balls drawn in this five random experiments?

2.(9%) In testing $H_0: \mu = 20$ versus $H_0: \mu > 20$ with $\alpha = 0.05$, based on a sample of $n = 30$, the sample mean and standard deviation were given as $\bar{x} = 23$ and $s = 5$.

(3%) (1) Calculate β for values of $\mu = 22$, $\mu = 23$, $\mu = 24$ and $\mu = 25$.

(3%) (2) Graph the power curve using the results of part (1).

(3%) (3) What would you expect to happen if the power curve moves from curve A to curve B? and what would be the stories if it moves from C to D ?

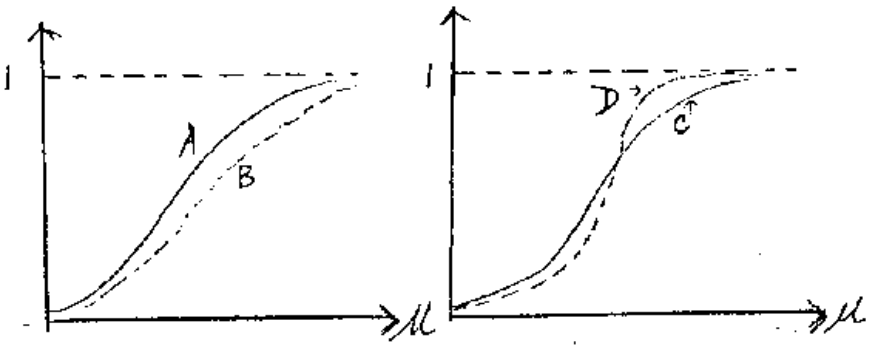
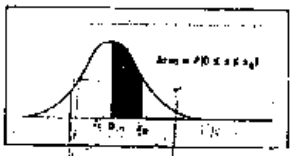


Table of normal curve areas



z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4263	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

This table is abridged from Table I of *Statistical Tables and Formulas*, by A. Hald (New York: John Wiley & Sons, Inc., 1952). Reproduced by permission of A. Hald and the publishers, John Wiley & Sons, Inc.

3. (13%) Given the following joint probability distribution

		X = 0	X = 1	X = 2
Y	0	0.05	0.1	0.03
	1	0.21	0.11	0.19
	2	0.08	0.15	0.08

(6%) (1) Calculate $Cov(X,Y)$ and $E(X^2Y^3)$, $E[Y - E(Y|X)]^2$

(7%) (2) Find the a and b that minimize the function $E[Y - a - bX]^2$. Given the solutions, verify that $E[Y - E[Y|X]]^2 \le E[Y - a - bX]^2$

4. (16 %) Answer the following questions as concisely as possible :

(4%) (1) Under what conditions the probability density function cannot be derived from its distribution function ? (Please point out two conditions rather than two examples.)

(4%) (2) Please give one exact example that the Central Limit Theory doesn't apply. Specifically, write down the probability density function of that example and the explanations.

(4%) (3) Is the probability density function of a continuous random variable continuous? Please give one example. Is the distribution function of a random variable surly continuous or at least continuous to the right ? Why is that ?

(4%) (4) What is the correct criterion to distinguish a random variable continuous or discontinuous? by its probability density function, distribution function or what?

5. (20%) Consider the following regression results for the estimated consumption function of the U.S. for the period 1980 to 1991(y = personal consumption expenditure, x = gross domestic product):

$$y = -231.7951 + 0.7194\,x$$
$$(94.5275) \quad (0.0217)$$

where the values in parentheses are the standard errors of the estimated coefficients.
(10%) (a) How do you interpret the coefficient 0.7194? Is this value significantly less than one?
(10%) (b) What is the F value ? Can you obtain the F value in terms of the coefficient of determination (R-square)?

6. (20%) Consider the following Cobb-Douglas equation for the agricultural sector of Taiwan for 1958-1972 (Y = real gross product, X2 = labor days, X3 = real capital input and ln is natural logarithm):

$$\ln Y = -3.3384 + 1.4988 \ln X2 + 0.4899 \ln X3$$
$$(2.4495) \quad (0.5398) \quad (0.1020) \qquad \text{R-square} = 0.8890$$

where the values in parentheses are the standard errors of the estimated coefficients. The covariance of estimates between ln X2 and ln X3 is -0.03842.
(10%)(a) Can you conclude that the Taiwanese agricultural sector was characterized by constant return to scale?

Suppose you obtain the regression results in the following way

$$\ln (Y / X2) = 1.7086 + 0.6129 \ln (X3 / X2)$$
$$(0.4159) \quad (0.0933) \qquad \text{R-square} = 0.8489$$

where the values in parentheses are the standard errors of the estimated coefficients.
(10%)(b) Can you conclude that the Taiwanese agricultural sector was characterized by constant return to scale? (Note: $F_{0.05}(1,12) = 4.75$, $F_{0.10}(1,12) = 3.18$)

7. (10%) A random variable X follows the poisson distribution if it has the following probability density function:

$$f(x) = \frac{e^{-\theta}\theta^{x}}{x!}$$

where θ is the parameter of the distribution. Using the maximum likelihood (ML) estimation method, show that the ML estimator of θ is $\hat{\theta} = \sum x_i / n$ where n is the sample size.

Table of the t Distribution

If X has a t distribution with n degrees of freedom, the table gives the value of x such that $Pr(X \le x) = p$.

n	p = .55	.60	.65	.70	.75	.80	.85	.90	.95	.975	.99	.995
1	.158	.325	.510	.727	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657
2	.142	.289	.445	.617	.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925
3	.137	.277	.424	.584	.765	.978	1.250	1.638	2.353	3.182	4.541	5.841
4	.134	.271	.414	.569	.741	.941	1.190	1.533	2.132	2.776	3.747	4.604
5	.132	.267	.408	.559	.727	.920	1.156	1.476	2.015	2.571	3.365	4.032
6	.131	.265	.404	.553	.718	.906	1.134	1.440	1.943	2.447	3.143	3.707
7	.130	.263	.402	.549	.711	.896	1.119	1.415	1.895	2.365	2.998	3.499
8	.130	.262	.399	.546	.706	.889	1.108	1.397	1.860	2.306	2.896	3.355
9	.129	.261	.398	.543	.703	.883	1.100	1.383	1.833	2.262	2.821	3.250
10	.129	.260	.397	.542	.700	.879	1.093	1.372	1.812	2.228	2.764	3.169
11	.129	.260	.396	.540	.697	.876	1.088	1.363	1.796	2.201	2.718	3.106
12	.128	.259	.395	.539	.695	.873	1.083	1.356	1.782	2.179	2.681	3.055
13	.128	.259	.394	.538	.694	.870	1.079	1.350	1.771	2.160	2.650	3.012
14	.128	.258	.393	.537	.692	.868	1.076	1.345	1.761	2.145	2.624	2.977
15	.128	.258	.393	.536	.691	.866	1.074	1.341	1.753	2.131	2.602	2.947
16	.128	.258	.392	.535	.690	.865	1.071	1.337	1.746	2.120	2.583	2.921
17	.128	.257	.392	.534	.689	.863	1.069	1.333	1.740	2.110	2.567	2.898
18	.127	.257	.392	.534	.688	.862	1.067	1.330	1.734	2.101	2.552	2.878
19	.127	.257	.391	.533	.688	.861	1.066	1.328	1.729	2.093	2.539	2.861
20	.127	.257	.391	.533	.687	.860	1.064	1.325	1.725	2.086	2.528	2.845