1. (20 %) Find the characteristic roots and vectors of the following matrix:

$$\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}.$$

2. (30 %) Suppose that the function $z = f(x, y)$ is homogeneous of degree 1. Show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z.$$

3. (30 %) Let $B$ be a closed convex set of points in $n$-dimensional Euclidean space, and let $x = (x_1, \ldots, x_n)$ be a point not in $B$. Prove that there exists numbers $p_1, \ldots, p_n, p_{n+1}$ such that $\sum_{i=1}^{n} p_i x_i = p_{n+1}$ and $\sum_{i=1}^{n} p_i y_i > p_{n+1}$, for all $y \in B$.

4. (20 %) Show that if a function $f$ is differentiable then it is also continuous.