

國立中央大學99學年度碩士班考試入學試題卷

所別：企業管理學系碩士班 一般甲組(一般生) 科目：微積分 共 2 頁 第 1 頁

本科考試禁用計算器

*請在試卷答案卷(卡)內作答

甲、選擇題：共 10 題，每題 5 分，共 50 分。請用大寫字母 A, B, C, D 或 E 答題，並將答案依題號順序寫在答案卷上。皆單選。

1. If $f'(0) = 1$, find $\lim_{h \rightarrow 0} \frac{f(-2h) - f(3h)}{h}$.
(A) 0 (B) 2 (C) -2 (D) 5 (E) -5
2. Which of the following series diverges?
(A) $\sum_{n=2}^{\infty} \frac{(\ln n)^2}{n}$ (B) $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ (C) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$ (D) $\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$ (E) $\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$
3. What is the value of the definite integral $\int_0^1 \frac{x}{\sqrt{4+5x}} dx$?
(A) $\frac{1}{5}$ (B) $\frac{14}{75}$ (C) $\frac{12}{75}$ (D) $\frac{2}{15}$ (E) none of these
4. Find the linearization of $f(x) = 3 + \int_1^{x^2} \sec(t-1) dt$ at $x = -1$.
(A) $L(x) = -3x + 2$ (B) $L(x) = 2x - 1$ (C) $L(x) = -2x + 1$ (D) $L(x) = 3x - 2$
(E) $L(x) = -3x + 1$
5. Find the limit: $\lim_{x \rightarrow 0^+} \frac{\int_0^{x^2} \sin \sqrt{t} - \sqrt{t} dt}{\int_0^{x^2} \tan \sqrt{t} - \sqrt{t} dt}$.
(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) -1
6. Evaluate $\int_0^1 \int_{y^2}^1 3ye^{x^2} dx dy$.
(A) $-\frac{3}{16} + \frac{3}{16}e$ (B) $\frac{3}{4} + \frac{3}{4}e$ (C) $-3e + 3e^2$ (D) $-\frac{3}{4} + \frac{3}{4}e$ (E) $-3 + 3e$
7. Find the surface area of the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 2$.
(A) $\frac{\sqrt{2}\pi}{2}$ (B) $\sqrt{2}\pi$ (C) $2\sqrt{2}\pi$ (D) $3\sqrt{2}\pi$ (E) $4\sqrt{2}\pi$
8. Find the length of the cardioid $r = 1 - \cos \theta$.
(A) 2π (B) 4π (C) 8 (D) 4 (E) 8π
9. Determine the maximum error when the fifth-degree Taylor polynomial is used to approximate e^{-x} in the interval $[0, \ln 2]$.
(A) $\frac{1}{6!}$ (B) $\frac{1}{2 \cdot 6!}$ (C) $\frac{1}{2 \cdot 5!}$ (D) $\frac{1}{5!}$ (E) none of these
10. Find the absolute maximum value of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) | 0 \leq x \leq 3, 0 \leq y \leq 2\}$.
(A) 16 (B) 9 (C) 4 (D) 1 (E) 0

注意：背面有試題

參考用

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所別：企業管理學系碩士班 一般甲組(一般生) 科目：微積分 共 2 頁 第 2 頁

本科考試禁用計算器

*請在試卷答案卷(卡)內作答

乙、填充題：共 5 題，每題 6 分，共 30 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$.

Answer : _____

2. If $f(x, y) = xe^{y^2}$, find the rate of change of f at the point $P(0, 2)$ in the direction from P to $Q(\frac{1}{2}, 2)$.

Answer : _____

3. Find the line that is tangent to the curve $x \sin 2y = y \cos 2x$ at $(\pi/4, \pi/2)$.

Answer : _____

4. Evaluate $\sum_{n=0}^{\infty} \int_n^{n+1} \frac{1}{1+x^2} dx$.

Answer : _____

5. A company expects its income during the next 5 years to be given by

$$c(t) = 100,000t, \quad 0 \leq t \leq 5.$$

Assuming an annual inflation rate of 10%, what is the present value of this income?

Answer : _____

丙、計算、證明題：共 2 題，每題 10 分，共 20 分。須詳細寫出演算過程，否則不予計分。

1. A cabinetmaker uses plantation-farmed mahogany to produce 5 furnishing each day. Each delivery of one container of wood is \$ 5000, whereas the storage of that material is \$ 10 per day per unit stored, where a unit is the amount of material needed by her to produce 1 furnishing. How much material should be ordered each time and how often should the material be delivered to minimize her average daily cost in the production cycle between deliveries?

2. A manufacturer's production is modeled by the Cobb-Douglas function $f(x, y) = 100x^{3/4}y^{1/4}$ where x represents the units of labor and y represents the units of capital. Each labor unit costs \$ 200 and each capital unit costs \$ 300. The total expenses for labor and capital cannot exceed \$ 60,000. Find the maximum production level.

參考用

注意：背面有試題