國立中央大學 109 學年度碩士班考試入學試題

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　　　 企業管理學系 碩士班 一般丙組(一般生)

科目： 統計學

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Part I Multiple-Choice Questions: (50 points)

*Identify the letter of the choice that **best** completes the statement or answers the question. There are 25 multiple-choice questions. 2 points for each question. **Please mark your answers in the multiple-choice answer card.***

1. Suppose the flights of STARLUX airlines depart on time with a probability of 0.85 according to the previous record. The airlines want to predict the delays among 8 flights that will depart tomorrow. Let X be the number of on-time flights, and Y be the number of delayed flights. Define P_1 and P_2 as follows.

    $P_1 = \Pr(X > 5)$

    $P_2 = \Pr(Y \le 3)$

    What is the relationship between P_1 and P_2?

    (A) $P_1 < P_2$

    (B) $P_1 = P_2$

    (C) $P_1 > P_2$

    (D) $P_1 + P_2 = 1$

2. X is a normal random variable with mean μ and standard deviation σ. k is a constant number. As the sample size increases, which one of the following statement is true about the probability $P(-k < \overline{X} < k)$, where $\overline{X}$ is the sample mean?

    (A) $P(-k < \overline{X} < k)$ increases.

    (B) $P(-k < \overline{X} < k)$ decreases.

    (C) $P(-k < \overline{X} < k)$ remains the same.

    (D) Cannot be determined.

3. Let A = $P(-1 < X < 1)$, B = $P(-1 < Y < 1)$, and C = $P(-1 < Z < 1)$, where $X \sim$ Normal($-1, 1^2$), $Y \sim$ Normal($0, 2^2$), and $Z \sim$ Normal($0, 1^2$). Choose the right inequality below:

    (A) A > B > C

    (B) C > A > B

    (C) C > B > A

    (D) A > C > B

4. X is a normal random variable with mean μ and standard deviation σ. Regarding the sample mean ($\overline{X}$), which one of the following statements is *not* true?

    (A) As the sample size increases, the standard deviation of $\overline{X}$ decreases.

    (B) As the sample size increases, the expected value of the sample mean remains the same.

    (C) As the sample size increases, the $100(1-\alpha)\%$ confidence interval of μ becomes narrower.

    (D) As the sample size increases, $P(\overline{X} > \mu)$ increases.

注意：背面有試題

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5. Suppose the population standard deviation is known. Which one of the following is *incorrect*?

   (A) The wider the margin of error is, the wider the confidence interval is.

   (B) The width of a confidence interval depends on the confidence coefficient.

   (C) The larger the sample mean is, the wider the confidence interval is.

   (D) When a sample is used to construct confidence intervals, the 98% confidence interval is always wider than the 95% confidence interval.

6. Regarding *t*-distribution, which one of the following is *incorrect*?

   (A) The mean can be negative.

   (B) When the degrees of freedom gets larger, the *t*-distribution approximates normal distribution.

   (C) *t*-distribution is symmetric.

   (D) When the degrees of freedom $n_1 > n_2$, the value of t_{α, n_1} is always smaller than the value of t_{α, n_2}.

7. Which of the following does not need to be known in order to compute the p-value?

   (A) The level of significance.

   (B) Knowledge of whether the test is one-tailed or two-tailed.

   (C) The value of the test statistic.

   (D) The distribution of the test statistics that should be used.

8. Using the same data set, if the null hypothesis is not rejected at the 1% level of significance, it

   (A) may be rejected or not rejected at the 5% level of significance

   (B) will always be rejected at the 5% level of significance

   (C) will never be rejected at the 5% level of significance

   (D) should never be tested at the 5% level of significance

9. Which following statement is **not** true for the difference of sample means $\bar{X}_1 - \bar{X}_2$?

   (A) $\bar{X}_1 - \bar{X}_2$ is a parameter because $\bar{X}_1$ and $\bar{X}_2$ are population parameters.

   (B) $\bar{x}_1 - \bar{x}_2$ is a estimate for $\mu_1 - \mu_2$.

   (C) $\bar{x}_1 - \bar{x}_2$ is always the middle point of the confident interval for $\mu_1 - \mu_2$.

   (D) $\bar{X}_1 - \bar{X}_2$ is a random variable with expected value $\mu_1 - \mu_2$.

10. A sample of 10 international locations was chosen and used to compare the calling rates per minute offered by CHT and TWM. Which of the following is correct?

    (A) The sampling is matched and the observations are paired.

    (B) The sampling is independent and the observations are paired.

    (C) The sampling is matched and the observations are not paired.

    (D) The sampling is independent and the observations are not paired.

參考用

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In questions 11 to 25, please try to identify the following notations in *simple linear regression model* into 4 categories, which are <u>given values</u>, <u>calculated values</u>, <u>parameters</u>, or <u>random variables</u>.

11. Independent variable, X, is a
    (A) given value;
    (B) calculated value;
    (C) parameter;
    (D) random variable.

12. Dependent variable, Y, is a
    (A) given value;
    (B) calculated value;
    (C) parameter;
    (D) random variable.

13. Coefficient in the regression model, β_1, is a
    (A) given value;
    (B) calculated value;
    (C) parameter;
    (D) random variable.

14. The least squared estimator, b_1, is a
    (A) given value;
    (B) calculated value;
    (C) parameter;
    (D) random variable.

15. Sampling error based on a particular sample, $|\beta_1 - b_1|$, is a
    (A) given value;
    (B) calculated value;
    (C) parameter;
    (D) random variable.

16. The first observation in a particular sample, (x_1, y_1), is a
    (A) given value;
    (B) calculated value;
    (C) parameter;
    (D) random variable.

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17. The error of the first observation (x_1, y_1), ε_1, is a
  (A) given value;
  (B) calculated value;
  (C) parameter;
  (D) random variable.

18. The residual of the first observation (x_1, y_1), e_1, is a
  (A) given value;
  (B) calculated value;
  (C) parameter;
  (D) random variable.

19. The predicted value of y for the first observation (x_1, y_1), $\hat{y}_1$, is a
  (A) given value;
  (B) calculated value;
  (C) parameter;
  (D) random variable.

20. The predicted value of y for a given value of x_p based on the least squared estimator, $\hat{y}_p$, is a
  (A) given value;
  (B) calculated value;
  (C) parameter;
  (D) random variable.

21. Prediction interval estimate, *P.I.E.*, is a
  (A) given value;
  (B) calculated value;
  (C) parameter;
  (D) random variable.

22. Test statistics of a *t*-test, *t*, is a
  (A) given value;
  (B) calculated value;
  (C) parameter;
  (D) random variable.

參考用

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23. Test value of a F-test for a particular sample, F, is a

　(A) given value;

　(B) calculated value;

　(C) parameter;

　(D) random variable.

24. Sum of squares due to the regression when <u>the sample is *not* specified</u>, *SSR*, is a

　(A) given value;

　(B) calculated value;

　(C) parameter;

　(D) random variable.

25. The coefficient of the determination to fit a particular dataset of x and y, r^2, is a

　(A) given value;

　(B) calculated value;

　(C) parameter;

　(D) random variable.

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Part II Workout Problems: (50 points)

1. (20 points) To estimate the population mean, μ, of a particular population, a ***random sample*** of size **5** is drawn. Let X_1, X_2, X_3, X_4, X_5, denote the observations. Andy, Betty, Cathy, David, and Eddy propose different estimators of μ as following:

   (A)ndy:　Sample mean, $\dfrac{X_1+X_2+X_3+X_4+X_5}{5}$;

   (B)etty:　Weighted sample mean, $\dfrac{X_1+2X_2+3X_3+4X_4+5X_5}{15}$;

   (C)athy:　Median, $X_{(3)}$;

   (D)avid:　Weighted sample mean by order statistics, $\dfrac{X_{(1)}+2X_{(2)}+3X_{(3)}+2X_{(4)}+X_{(5)}}{9}$;

   (E)ddy:　Truncated sample mean, $\dfrac{X_{(2)}+X_{(3)}+X_{(4)}}{3}$.

Please compare these proposed estimators and **Rank** them based on unbiasedness, efficiency, consistency, and etc. Try to show your idea to support your comparison as much as possible, (for example, sufficient statistics,… etc.)

2. Following (1), assume the population follows a normal distribution. Frank has drawn a sample and provided a 95% confidence interval of (75, 95), Grace has drawn another sample and provided a 95% confidence interval of (72, 88), and Hector has drawn another sample and provided a 95% confidence interval of (73, 95).

   (a) (5 points) What is the probability that **all** these three intervals cover the parameter μ? Assume μ has a prior distribution of normal with mean 83 and variance 25.

   (b) (10 points) Let's pool all observations of these three samples together, and please provide a pooled 95% confidence interval of μ. What is the probability of this pooled confidence interval covers μ.

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3. Assume that the birthweight in grams of a baby born in Taiwan is $N(3315, 525^2)$, boys and girls combined. Let X be the weight of a <u>baby girl</u> who is born at home in ChungLi city and assume that the distribution of X is $N(\mu_X, \sigma_X^2)$.

(a) (5 points) Using 11 observations of X, **define a test statistic** and **critical region**, $\alpha = 0.01$, for testing $H_0: \mu_X = 3315$ against the alternative hypothesis $H_a: \mu_X > 3315$ (home-born babies are heavier).

(b) (5 points) Calculate the value of the test statistic and give your conclusion using the following weights:

3120　2657　3459　3629　3345　3629　3515　3856　3629　3345　3062

(c) (5 points) Let's assume the standard deviation of X is the same as the standard deviation of birthweight of a newborn baby in Taiwan. Please redo (a) and (b).
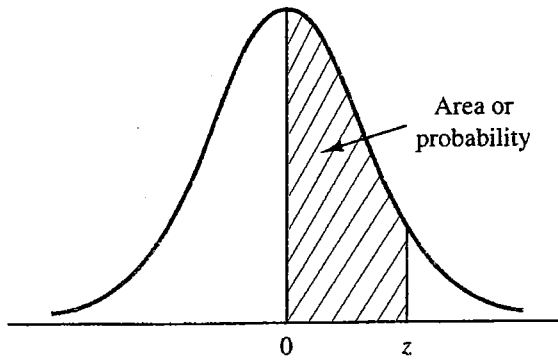
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Area or probability

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean and z is .3944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

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參考用

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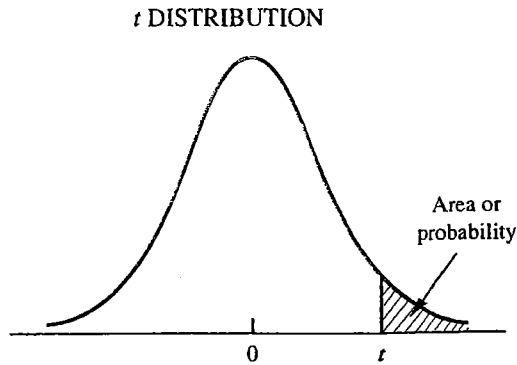
Area or probability

0　　t

Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail				
	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

參考用