所別：企業管理學系碩士班 一般甲組(一般生)　科目：工程數學　共 1 頁 第 1 頁

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

一、**(20%)** Please use Laplace transform to solve the following initial value problem:

$y''(t)-6y'(t)+9y(t)=0$, with $y(0)=2$ and $y'(0)=4$.

二、**(20%)** Please answer the following question:

(一) (10%) Considering a matrix $A=\begin{bmatrix}10 & 4\\ 3 & 6\end{bmatrix}$, for what values of constant k does matrix $A-k\cdot I=\begin{bmatrix}10-k & 4\\ 3 & 6-k\end{bmatrix}$ fail to be invertible? The matrix **I** is the 2-by-2 identity matrix.

(二) (10%) For a matrix $B=\begin{bmatrix}9 & 5\\ -2 & 2\end{bmatrix}$, please find a nonzero vector in the kernel (null space) of matrix **B**-7·**I**, where **I** is the 2-by-2 identify matrix.

三、 **(20%)** Please find the general solution of the following ordinary differential equation (ODE):

$$y'''(x)-y''(x)-17\cdot y'(x)-15\cdot y(x)=18\cdot e^{-x}$$

四、**(20%)** For a continuous-time non-periodic signal $x(t)$, the Fourier spectrum of $x(t)$ is :

$$X(j\omega)=\begin{cases}1, for\ \omega\le|\omega_c|\\ 0, else\end{cases},$$

where ω_c is the cut-off frequency. Please derive its inverse Fourier transform.

五、**(20%)** Please prove the following relations:

(一) (10%) $F\{x(t)*g(t)\}=X(j\omega)\cdot G(j\omega)$,

where $F\{\cdot\}$ is the Fourier transform operator, $X(j\omega)$ and $G(j\omega)$ are the Fourier transforms of $x(t)$ and $g(t)$, respectively.

(二) (10%) $L\{f^{(n)}(t)\}=s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)-\cdots-f^{(n-1)}(t)$,

where $L\{\cdot\}$ is the Laplace transform operator, $f^{(n)}(t)$ is the n^{th} derivative of $f(t)$, and $F(s)$ is the Laplace transform of $f(t)$.