

國立中央大學九十一年度碩士班研究生入學試題卷

所別: 水文科學研究所 不分組 科目: 應用數學 共 1 頁 第 1 頁

中央大學 91 學年度碩士班研究生入學試題
水文科學研究所 不分組 應用數學

- Find the general solution of the following equations. (15%)
 - $y' - e^x y^2 = 0$
 - $xy' - y = x$
 - $y'' - 8y' + 16y = 0$
- Find solutions of the following problems. (10%)
 - $xy' + y = 0, y(1) = 1$
 - $8y'' + 6y' + y = 0, y(-2) = 3e, y'(-2) = -\frac{3}{2}e$
- Find the homogeneous solution of the differential equation. (15%)
$$x^2 y'' - 3xy' + 4y = x^2$$
- Find the directional derivative of $f = 2x^2 + xy + yz^2$ at $(1, -1, 2)$ in the direction of the vector $A = \hat{i} - 2\hat{j} + 2\hat{k}$. (15%)
- Applying the divergence theorem to evaluate $\int_S (x^3 dydz + x^2 y dzdx + x^2 z dx dy)$, where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2, 0 \leq z \leq b$ (15%)
- Are the following functions harmonic? If so, find a corresponding analytic function $f(z) = u(x, y) + v(x, y)i$. (10%) [Hint: Using Cauchy-Riemann equations]
 - $u = y^2 - x^2$
 - $v = (x^2 - y^2)^2$
- Do the following sets of equations have a solution? Find a solution if it exists. (10%)
 - $$\begin{cases} 3x - 2y + z = 2 \\ x - 3y + z = 5 \\ x + y - z = -5 \\ 3x + 0y + z = 0 \end{cases}$$
 - $$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 4 \end{bmatrix}$$
- Find the eigenvalues and eigenvectors of the following matrix. (10%)
$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$