

(1) Sketch the graphs of $f(x) = \frac{\sqrt{x^2+1}}{x}$ and $x^2y^2 - y^2 + x^2 = 0$. 10%

(2) Find $\int_1^4 (t\sqrt{t} + \frac{3}{t^2}) dt$ and $\int_{-1}^8 x^{-2/3} dx$. 10%

(3) Find the area of the region bounded by the graphs of

a) $f(x) = \sqrt{x}$ and $g(x) = x^2$

b) $y^2 = x+1$ and $x+y=1$ 10%

(4) Find $\cos[\arccos\frac{5}{13} + \arcsin\frac{3}{5}]$. 10%

(5) Evaluate $\int \sqrt{4-x^2} dx$. 10%

(6) Find the length of the cardioid $r = 2(1+\cos\theta)$. 10%

(7) Find the tangential and centripetal components of acceleration of a particle whose motion is given by

$$\vec{r}(t) = t\hat{i} + t\hat{j} + t^2\hat{k} \quad 0 \leq t \leq 1. \quad 10\%$$

(8) Solve the following differential equations 30%

a) $\frac{dy}{dx} + y = xy^3$

b) $y'' - 2y' + 10y = 0, y(\pi/2) = 0, y'(\pi/2) = 1$

c) $y''' - 4y'' + 5y' - 2y = 0, y(0) = 0, y'(0) = 1, y''(0) = 0.$