所別： 地球科學學系地球物理 碩士班 不分組(一般生)　　　

　　　 地球科學學系地球物理 碩士班 不分組(在職生)

科目： 微積分

＊本科考試禁用計算器

作答時須列出完整計算過程，無計算過程者不予計分

1. (a) (5%) $\lim_{x \to 1} \dfrac{x-1}{\sqrt{x^2+3}-2} = ?$

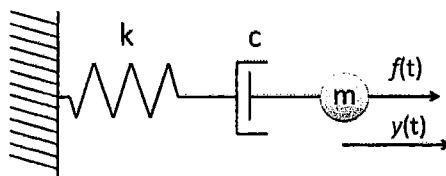   (b) (5%) $y = ln\left(\dfrac{x-1}{x+1}\right), \quad find \left(\dfrac{dy}{dx}\right)?$

2. (a) (5%) $\dfrac{d}{dx}\left(\int_{A(x)}^{B(x)} e^t dt\right) = ?$

   (b) (5%) $\int \dfrac{1}{\sqrt{a^2-x^2}} dx = ?$

3. (a) (5%) $\int x^2 cos 5x \, dx = ?$

   (b) (5%) $\int_{-\infty}^{\infty} e^{-x^2} dx = ?$

4. (a) (5%) Derive the ordinary differential equation for the mass-spring-damper system as shown in the figure, where k is the spring constant, m is the mass of the ball, c is the damping constant, *y(t)* is position of the ball, and *f(t)* is the input force.

   (b) (5%) Solve for *y(t)* in the case where c = 0 and *f(t)* = 0.

5. (10%) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$.

   (hint: one of the eigenvalues is λ = 4.)

注意：背面有試題

國立中央大學 113 學年度碩士班考試入學試題

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6. (5%) (a) Show that $\mathrm{curl}(\mathrm{grad}\, f) = 0$ for a scalar function f.

　　(5%) (b) Show that $\mathrm{div}(\mathrm{curl}\, \vec{v}) = 0$.

7. (10%) 利用三重積分求半徑為 a 之球的體積。

8. (10%) Find a general solution.

$$y'' + 10y' + 25y = e^{-5x}$$

9. (10%) Use the method of separating variables to solve the one-dimensional wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$, for the vibrations of an elastic string of length L.
   The boundary conditions are $u(0, t) = 0$, $u(L, t) = 0$ for all t.
   The initial conditions are $u(x, 0) = f(x)$, $u_t(x, t)|_{t=0} = 0$.

10. (10%) Fermat's principle states that the path taken by a ray of light between two points is the least-time path.
    Derive Snell's law using Fermat's principle.

注意：背面有試題