

國立中央大學 106 學年度碩士班考試入學試題

所別：地球科學學系地球物理 碩士班 不分組(一般生)
地球科學學系地球物理 碩士班 不分組(在職生)

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科目：微積分

本科考試禁用計算器

*請在答案卷 內作答

作答時須列出完整計算過程

參考用

1. (a) $\lim_{x \rightarrow \infty} (\ln x)^{\frac{1}{x}} = ?$ (5%)

(b) $\lim_{x \rightarrow \infty} \frac{\sin(\frac{1}{x})}{\tan^{-1}(\frac{1}{x})} = ?$ (5%)

2. (a) $y = \sin^{-1}\left(\frac{x}{2}\right)$, $|x| < 2$, $\frac{dy}{dx} = ?$ (5%)

(b) $f(x) = \frac{1}{x}$ and $x \neq 0$, $f^{(n)}(x) = ?$ (5%)

3. (a) $\int \frac{(x-3)dx}{x^3+3x^2+2x} = ?$ (5%)

(b) $\int e^{ax}(\cos bx)dx = ?$ (5%)

4. (10%) Find the odd periodic expansions of the function (half-range expansion)

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L. \end{cases}$$

5. (10%) Use Laplace transform to solve

$$y'' + y = 2t, \quad y\left(\frac{1}{4}\pi\right) = \frac{1}{2}\pi, \quad y'\left(\frac{1}{4}\pi\right) = 2 - \sqrt{2}.$$

注意：背面有試題

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6. Use the method of separating variables to solve the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, for the vibrations of an elastic string of length L . The boundary conditions are $u(0, t) = 0$ and $u(L, t) = 0$ for all t . The initial conditions are $u(x, 0) = f(x)$ and $\left. \frac{\partial u}{\partial t} \right|_{t=0} = g(x)$ (10%)

7. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ (10%)

8. Find a general solution of $y'' + 4y' + 3y = 4.5\sin 2t$ (10%)

9. Find a general solution of $xy' = 2y + x^3 e^x$ (10%)

10. Use the divergence theorem to evaluate the surface integral $\iint_S \vec{F} \cdot \hat{n} dA$, where $\vec{F} = [x^2, 0, z^2]$, \hat{n} unit normal vector, S the surface of the box $|x| \leq 1, |y| \leq 3, |z| \leq 2$. (10%)