1. Let the isotherms (=curves of constant temperature) in a body in the upper half-plane $y > 0$ be given by $4x^2 + 9y^2 = c$. Find their orthogonal trajectories.

   (10%)

2. Solve the following initial value problem
   $$y'' + 6y' + 8y = 4\sin 2t, \quad y(0) = 0.7, \quad y'(0) = -11.8$$

   (15%)

3. Find eigenvalues and eigenfunctions of the following problem
   $$y'' + 8y' + (\lambda + 16)y = 0, \quad y(0) = 0, \quad y(\pi) = 0$$

   (15%)

4. Using Laplace transforms, solve the following integral equation

   $$y(t) + 2e^t \int_0^t e^{-\tau} y(\tau)d\tau = te^t$$

   (15%)

5. Find the eigenvalues and eigenvectors of the following matrix.

   $$\begin{bmatrix} 13 & 5 & 2 \\ 2 & 7 & -8 \\ 5 & 4 & 7 \end{bmatrix}$$

   (10%)

6. Let $f = zy + yx$, $\mathbf{v} = \begin{bmatrix} y & z & 4z-x \end{bmatrix}$, $w = \begin{bmatrix} y^2 & z^2 & x^2 \end{bmatrix}$. Find

   a. ∇f　　　　　b. $\nabla \cdot \mathbf{v}$
   c. $\nabla \times \mathbf{w}$　　　d. $\nabla^2 f^2$
   e. $\nabla \cdot (\mathbf{v} \times \mathbf{w})$

   (15%)

7. Find the Fourier series of the following function
   $$f(x) = \begin{cases} x & \text{if } -\pi/2 < x < \pi/2 \\ \pi - x & \text{if } \pi/2 < x < 3\pi/2 \end{cases}$$

   (10%)

8. Represent $f(x,y)$ ($0<x<a$, $0<y<b$) by a double Fourier series.
   $$f(x,y) = xy(a^2 - x^2)(b^2 - y^2)$$

   (10%)

參考用