

國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 大氣物理研究所 不分組 科目: 應用數學 共 / 頁 第 / 頁

1. Solve the initial value problem:

(15 %)

$$\frac{d^2y}{dt^2} + 2y = r(t)$$

with the initial conditions of $y(0) = 0$ and $\frac{dy}{dt}(0) = 0$ where

$$r(t) = \begin{cases} 1 & \text{if } 0 < t < 1 \\ 0 & \text{if } t \geq 1 \end{cases}$$

2. Find the Laplace transform of the given function

(10 %)

$$f(t) = \begin{cases} t & \text{if } 0 < t < \pi \\ t - \pi & \text{if } \pi < t < 2\pi \end{cases}$$

and

$$f(t) = f(t + 2\pi) \text{ for } t > 0$$

3. (a) Show that all the eigenvalues of a symmetric matrix are real.

(10 %)

(b) Show that the inverse of an orthogonal matrix is orthogonal.

(5 %)

4. A coupled first-order system is given by of $y' = Ay + g e^{-4t}$ where

$$y = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \quad A = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \text{ and } g = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$$

(a) Find all the eigenvalues and the corresponding eigenvectors of A.

(5 %)

(b) Find the general solution using the method of diagonalization

(15 %)

(i.e., decouple the system by diagonalization).

5. Let $V = xy \hat{i} + (y - z)^2 \hat{j} + 2xyz \hat{k}$, please find $\text{div}(\text{curl } V)$, i.e., $\nabla \cdot (\nabla \times V)$.

(10 %)

6. Show that using Fourier transform the general solution of the wave equation

(15 %)

$$u_{tt} = c^2 u_{xx} \quad (-\infty < x < \infty, t > 0)$$

with the following conditions

$$u(x, 0) = f(x) \quad (\text{initial deflection}),$$

$$u_t(x, 0) = 0 \quad (\text{initial speed zero}), \text{ and}$$

$$u \rightarrow 0, u_x \rightarrow 0 \quad \text{as } |x| \rightarrow \infty \text{ for all } t$$

is D'Alembert's solution given by $u(x, t) = \frac{1}{2}[f(x - ct) + f(x + ct)]$

where c is a real constant.

7. Evaluate the integral $\int_{-\infty}^{\infty} \frac{2}{x^4 - 1} dx$.

(15 %)

