

國立中央大學100學年度碩士班考試入學試題卷

所別：大氣物理研究所碩士班 不分組(一般生) 科目：應用數學 共 / 頁 第 / 頁

本科考試禁用計算器

*請在試卷答案卷(卡)內作答

參考用

1. Explain and compare the following terms.
 - a. Linear and nonlinear differential equations;
 - b. Free and forced oscillations;
 - c. Laplace and Fourier transforms;
 - d. Fourier series and Fourier integral;
 - e. Stokes and divergent theorems

(15%)

2. Solve the following problems.

- a. $y' + (x+1)y = e^{x^2}y^3, \quad y(0) = 0.5$
- b. $(x^{-1}y)' + (\lambda+1)x^{-3}y = 0, \quad y(1) = 0, \quad y(e^\pi) = 0$
- c. $y'' + 2y' + 26y = 13 \cos 3t, \quad y(0) = 1, \quad y'(0) = 0.4$

(30%)

3. Find the inverse Laplace transform of the following functions

- a. $\frac{5s+4}{s^2}e^{-2s},$
- b. $\frac{\pi}{s^2(s^2+\omega^2)}$

(15%)

4. Let $f = xy - yz, \mathbf{v} = [2y \ 2z \ 4x+z], \mathbf{w} = [3z^2 \ 2x^2 - y^2 \ y^2]$. Find

- a. ∇f
- b. $\nabla \cdot \mathbf{v}$
- c. $\nabla \times \mathbf{w}$
- d. $\nabla \cdot (\nabla f)$
- e. $\nabla \cdot (\mathbf{v} \times \mathbf{w})$

(15%)

5. Let $\mathbf{A} = \begin{bmatrix} 9 & 2 & 8 \\ 2 & 18 & 10 \\ 8 & 10 & 15 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 2 & 6 \\ -2 & 0 & -3 \\ -6 & 3 & 0 \end{bmatrix}, \mathbf{a} = \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

please calculate $\mathbf{AB}, \mathbf{A}^2 + \mathbf{B}^2, \mathbf{a}^T \mathbf{A} \mathbf{a}, \mathbf{a}^T \mathbf{b}, \det(\mathbf{AB})$

(15%)

6. Solve the one-dimensional heat equation,

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2},$$

that satisfies $u_x(0,t) = u_x(L,t) = 0$ and $u(x,0) = f(x)$

(10%)