國立中央大學100學年度碩士班考試入學試題卷

所別：環境工程研究所碩士班 甲組(一般生)　科目：工程數學　共 1 頁 第 1 頁
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本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

1) There are two points A and B in the plane. The coordinates of A are $(x,y)=(1,0)$ while those of B are $(x,y)=(0,1)$. A path Γ connects A and B, and it is described by the equation $y=(x-1)^2$. Let $P=1+y-\dfrac{x}{x^2+y^2}$ and $Q=x-\dfrac{y}{x^2+y^2}$, please compute the path integral $J=\int_{\Gamma} P\,dx+Q\,dy$ along Γ from point A to point B.

(25%)

2) Let $A=(A_{ij})=\begin{pmatrix}-9 & -4 & 1\\ -27 & -6 & 15\\ -27 & 12 & -3\end{pmatrix}$ be a 3×3 matrix. If λ_1, λ_2 and λ_3 are its eigen-values, please find out the sum of these three eigen-values. That is, please compute the value of $\lambda_1+\lambda_2+\lambda_3$.

(25%)

3) A particular solution $y(t)$ of the differential equation $\dfrac{d^2y}{dt^2}+4y=\cos(2t)$ satisfies the initial conditions $y(0)=1$ and $\dfrac{dy}{dt}(0)=0$. That is, $y=1$ and $\dfrac{dy}{dt}=0$ at the initial time $t=0$. Please compute the value of $y(t)$ at the time $t=4$.

(25%)

4) A particular solution $y(x)$ of the differential equation $x\dfrac{dy}{dx}+y=xe^x$ satisfies the condition $y(1)=0$. That is, $y=0$ at the point $x=1$. Please compute the value of $y(x)$ at the point $x=2$.

(25%)