系所別: 機械工程學系 甲組乙組丙組己組 科目: 工程數學

參考用

1. Let

$$\mathbf{A}=\begin{bmatrix} 1 & 2+i & 3-2i & 4+3i & 5-4i \\ 2-i & 2 & 4-3i & 5+4i & 6-5i \\ 3+2i & 4+3i & 3 & 6+5i & 7-6i \\ 4-3i & 5-4i & 6-5i & 4 & 2 \\ 5+4i & 6+5i & 7+6i & 2 & 5 \end{bmatrix}_{5\times 5}$$

Prove that all eigenvalues of $\mathbf{A}$ are real. (6%)

2. For the linear system of equations $\mathbf{Ax}=\mathbf{b}$, where

$$\mathbf{A}=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & k_1 \\ 3 & k_2 & 0 \\ 4 & 5 & 10 \end{bmatrix},\ \mathbf{x}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},\ \text{and}\ \mathbf{b}=\begin{bmatrix} 1 \\ b_2 \\ 3 \\ 4 \end{bmatrix},$$

(a) determine the values of k_1, k_2, and b_2, for which the system has infinitely many solutions; (4%)

(b) determine the values of k_1, k_2, and b_2, for which the system has precisely one solution with $x_3 \neq 0$; (4%)

(c) determine the values of k_1, k_2, and b_2, for which the system has precisely one solution with $x_1 = 1$. (4%)

3. Let $\mathbf{D}=\mathbf{x}^{-1}\mathbf{Ax}$ be diagonal, with the eigenvalues of $\mathbf{A}$ as the entries on the main diagonal.

(a) Prove that

$$\mathbf{D}^m=\mathbf{x}^{-1}\mathbf{A}^m\mathbf{x} \qquad (m=2,3,\ldots) \qquad (3\%)$$

(b) Find $\mathbf{A}^{10}$ where $\mathbf{A}=\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. (4%)

4.

(a) Solve the problem

$$\frac{dy}{dx}=\frac{-\sin y}{(1+x)\cos y} \qquad (4\%)$$

(b) Solve the problem

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0 \qquad (5\%)$$

(c) Find the general solution to the equation

$$\frac{d^2y}{dx^2}+y=\cos x \qquad (6\%)$$

(d) What is the relationship between the Fourier transformation and the Laplace transformation? Can the function $f(x)=x3^x$ be transformed by the two methods? Explain. (10%)

注意：背面有試題

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5.

(a) Show that the vector field $\vec{F} = (x+2y)\vec{i} + (2x-y)\vec{j}$ is a gradient field. Find a potential function for $\vec{F}$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, C: (1,0) to (3,2). (6%)

(b) Evaluate the line integral

$$\oint_C \frac{-y^3 dx + xy^2 dy}{(x^2+y^2)^2}, \text{ where } C \text{ is the ellipse } x^2 + 4y^2 = 4. \quad (9\%)$$

(c) Use Stokes's theorem to evaluate

$$\oint_C z^2 e^{x^2} dx + xy^2 dy + \tan^{-1} y dz,$$

where C is the circle $x^2 + y^2 = 9$, by finding a surface S with C as its boundary and such that the orientation of C is counterclockwise. (10%)

6.

(a) Use separation of variables to find product solutions of $\dfrac{\partial^2 u}{\partial x^2} = 9\dfrac{\partial u}{\partial y}$. (10%)

(b) Use the Laplace transform to solve the boundary-value problem (15%)

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < 1, \quad t > 0,$$

$$u(0,t) = 0, \quad u(1,t) = 0,$$

$$u(x,0) = 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 2\sin \pi x + 4\sin 3\pi x.$$