

國立中央大學九十一學年度碩士班研究生入學試題卷

所別: 機械工程學系 丁組 科目: 工程數學 共 2 頁 第 1 頁

1. (a) Solve the initial value problem

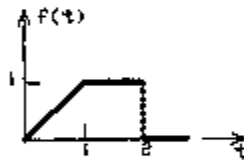
$$ay' + by = 0, \quad y(0) = 0, \quad y'(0) = 1,$$

where a and b are constants, but $a \neq 0$. (12%)

(b) Find a basis of solution of the differential equation. (Show the details of your work.) (8%)

$$x^2 y'' + 3xy' + y = 0$$

(c) Find the Laplace transforms of the following function. (Show the details of your work.) (5%)



2. (a) Evaluate $\oint_c \frac{e^z}{(z-1)(z+4)} dz$, where c is the circle $|z|=3$ described in the positive direction. (8%)

(b) Evaluate $\oint_c z^6 \sin(1/z) dz$, where c is the circle $|z|=1$ described in the positive direction. (7%)

(c) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5-4\cos\theta} d\theta$. (10%)

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3. (a) Find the similarity transformation $A = PAP^{-1}$, where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and Λ is a diagonal matrix. (10%)

(b) Consider a system of differential equations $\frac{dy}{dx} = Ay$ subject to the initial condition

$y(0) = b$, where the matrix A is given as above, $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. We can solve this

problem by taking the iterative procedure:

$$y^{(0)} = b,$$

$$y^{(1)} = b + \int_0^x Ay^{(0)} d\xi = b + xAb,$$

$$y^{(2)} = b + \int_0^x Ay^{(1)} d\xi = b + xAb + \frac{(xA)^2}{2!} b,$$

⋮

$$y^{(n)} = \left[I + \frac{xA}{1!} + \frac{(xA)^2}{2!} + \dots + \frac{(xA)^n}{n!} \right] b,$$

and $y^{(n)} \rightarrow y$ as $n \rightarrow \infty$. Obtain y_1 and y_2 by the iteration method and the similarity transformation you have got. Show the details of your work. (Hint: think about the Taylor series expansion for e^t about $t=0$.) (15%)

4. 請寫一詳細的程式流程圖，讓程式可自動猜測使用者所選定的整數值(此數值介於 0 至 100)，直到猜中為止。方式是程式可詢問使用者其所選定的整數值是否比程式所猜的整數值來的大，還是小，或相等(相等即表示猜中，程式結束。需輸出猜測的次數)。 (25%)

註：程式執行步驟及猜值的方法必須有效率，評分重點。例如不能按順序一一測試詢問。
舉例：假設使用者所選定的整數值是 90，程式猜 40，則詢問使用者其整數值是‘大於’或‘小於’或‘相等’於 40，使用者回答‘大於’之後，程式據此新猜一整數值，例如 65，則使用者再次回答‘大於’。如此類推並持續到猜中為止。