國立中央大學九十一學年度碩士班研究生入學試題卷

所別：機械工程學系甲.乙.丙.己組　科目：工程數學　共 1 頁 第 1 頁

1. (a) Solve the initial value problem

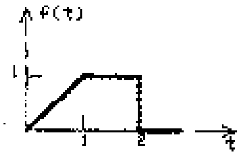
$$ay'' + by = 0, \qquad y(0) = 0, \quad y'(0) = 1,$$

where a and b are constants, but $a \neq 0$. (12%)

(b) Find a basis of solution of the differential equation. (Show the details of your work.) (8%)

$$x^2 y'' + 3xy' + y = 0$$

(c) Find the Laplace transforms of the following function. (Show the details of your work.) (5%)



2. (a) Evaluate $\oint_c \frac{e^z}{(z-1)(z+4)}\, dz$, where c is the circle $|z| = 3$ described in the positive direction. (8%)

(b) Evaluate $\oint_c z^6 \sin(1/z)\ dz$, where c is the circle $|z| = 1$ described in the positive direction. (7%)

(c) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 - 4\cos\theta}\, d\theta$. (10%)

3. (a) Find the similarity transformation $A = P\Lambda P^{-1}$, where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and Λ is a diagonal matrix. (10%)

(b) Consider a system of differential equations $\frac{dy}{dx} = Ay$ subject to the initial condition $y(0) = b$, where the matrix A is given as above, $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. We can solve this problem by taking the iterative procedure:

$$y^{(0)} = b,$$

$$y^{(1)} = b + \int_0^x Ay^{(0)} d\xi = b + xAb,$$

$$y^{(2)} = b + \int_0^x Ay^{(1)} d\xi = b + xAb + \frac{(xA)^2}{2!} b,$$

$$\vdots$$

$$y^{(n)} = \left[I + \frac{xA}{1!} + \frac{(xA)^2}{2!} + \cdots + \frac{(xA)^n}{n!} \right] b,$$

and $y^{(n)} \to y$ as $n \to \infty$. Obtain y_1 and y_2 by the iteration method and the similarity transformation you have got. Show the details of your work. (Hint: think about the Taylor series expansion for e^t about $t = 0$.) (15%)

4. By the method of separation of variables, find the solution $u(x, y)$ of the Poisson equation

$$u_{xx} + u_{yy} = \cos(\pi y),$$

in the semi-infinite strip $0 \le x < \infty, 0 \le y \le 1$, such that

$$u(0, y) = y, \quad u_y(x, 0) = u_y(x, 1) = 0.$$ (25%)

參考用