國立中央大學九十學年度碩士班研究生入學試題卷

所別：化學工程與材料工程系　科目：工程數學　共 2 頁 第 1 頁

(1) (10%+10%)

Find a general solution for a non-homogeneous linear system of ordinary differential equations:

$$\mathbf{y}' = \mathbf{A}\mathbf{y} + \mathbf{g} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \mathbf{y} + \begin{bmatrix} -6 \\ 2 \end{bmatrix} e^{-2t}$$

by undetermined - coefficient method **and** by Method of Diagonalization.

(2) (15%)

Solve the initial value problem $y'' - y = t$, $y(0) = 1$ and $y'(0) = 1$ by Laplace Transform.

(3) (10%)

Consider the function $f(x,y,z) = 4(x^2 + y^2) - z^2$. The surface defined by $f(x,y,z) = 0$ is a cone. Find the unit normal vector on the surface of the cone $f(x,y,z) = 0$ at the point P, whose coordinate is (1, 0, 2).

(4) (10%)

Compute the line integral $\int_C \mathbf{F}(\mathbf{r}) \bullet d\mathbf{r}$ where $\mathbf{F}(\mathbf{r}) = 5z\mathbf{i} + xy\mathbf{j} + x^2 z\mathbf{k}$ and the path C is a parabolic with $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$, $0 \le t \le 1$.

(5) (10%)

Find the eigenvalues and eigenvectors for the following matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

(6) (7.5%)(是非題)

A periodic function $u(t) = u(t + \frac{2\pi}{\omega})$ can be written as Fourier series $u(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$.

Which of the following statement is correct.

Prob.	If for $-\frac{\pi}{\omega} \le t \le \frac{\pi}{\omega}$	then
1	$u(t) = \begin{cases} 0 & \text{if } -\pi/\omega < t < 0 \\ 3\sin\omega t & \text{if } 0 < t < \pi/\omega \end{cases}$	$a_0 = 0$, $b_n = 0$ for $n \ge 2$
2	$u(t) = (t + \pi/\omega)^2$	$a_n = 0$ for $n \ge 1$
3	$u(t) = t^2$	$b_n = 0$, for $n \ge 1$
4	$u(t) = e^{-2t}$	$a_n \ne b_n \ne 0$
5	$u(t) = e^{-\lvert t \rvert}$	$a_n = 0$

(7)(20%)(是非題)

Which of the following PDE plus boundary (initial) conditions set leads to analytical solution.

Prob.	PDE	Range	BC	IC
1	$\frac{\partial^2 u}{\partial t^2}=c^2\nabla^2 u$	$u(x,t)$, $0\le x\le L$	$u(0,t)=A$ $u(L,t)=B$	$u(x,0)=f(x)$ $\dot{u}(x,0)=g(x)$
2	$\frac{\partial^2 u}{\partial t^2}=c^2\nabla^2 u$	$u(x,t)$, $0\le x\le L$		$u(x,0)=f(x)$ $\dot{u}(x,0)=g(x)$
3	$\frac{\partial^2 u}{\partial t^2}=c^2\nabla^2 u$	$u(x,t)$, $0\le x\le L$	$u(0,t)=f(t)$ $u(L,t)=0$	$u(x,0)=0$ $\dot{u}(x,0)=0$
4	$\frac{\partial u}{\partial t}=c^2\nabla^2 u$	$u(x,t)$, $0\le x\le \infty$		$u(x,0)=f(x)$
5	$\frac{\partial^2 u}{\partial t^2}=c^2\nabla^2 u$	$u(x,y,t)$ $0\le x\le L,\ 0\le y< L$	$u(0,y,t)=0,\ u(x,0,t)=A$ $u(L,y,t)=0,\ u(x,L,0)=0$	$u(x,0)=f(x)$ $\dot{u}(x,0)=g(x)$
6	$\frac{\partial^2 u}{\partial t^2}=c^2\nabla^2 u$	$u(r,t)$ $0\le r\le R$	$u(R,t)=$ Const.	$u(r,0)=f(r)$ $\dot{u}(r,0)=g(r)$
7	$\frac{\partial u}{\partial t}=c^2\nabla^2 u$	$u(x,t)$, $0\le x\le L$	$a_1\cdot u(0,t)+b_1\cdot u'(0,t)=0$ $a_2\cdot u(L,t)+b_2\cdot u'(L,t)=0$	$u(x,0)=f(x)$
8	$\nabla^2 u=0$	$u(x,y)$ $0\le x\le 1,\ 0\le y\le 1$	$a_1\cdot u(0,t)+b_1\cdot u'(0,t)=0$ $a_2\cdot u(1,t)+b_2\cdot u'(1,t)=0$	
9	$\nabla^2 u=0$	$u(r,\theta,\phi)$ $0\le r\le\infty$	$u(\infty,\theta,\phi)=0$	
10	$\nabla^2 u=0$	$u(r,\theta,\phi)$ $R\le r\le\infty$	$u(R,\theta,\phi)=f(\phi)$, $u(\infty,\theta,\phi)=0$	

(8)(7.5%) (多重選擇題)

(8.1)The analytical solution of $\nabla^2 u=0$ in spherical coordinate system may involve (multiple choice):
(a) Legendre polynomial (b) Bessel function (c) Fourier series (d) Laguerre polynomial

(8.2)The analytical solution of $\nabla^2 u=0$ in cylindrical coordinate system may involve (multiple choice):
(a) Legendre polynomial (b) Bessel function (c) Fourier series (d) Laguerre polynomial

(8.3) The analytical solution of $\nabla^2 u=0$ in Cartesian coordinate system may involve (multiple choice):
(a) Legendre polynomial (b) Bessel function (c) Fourier series (d) Laguerre polynomial