注意：本試題共 6 題，每題 20 分，任選 5 題，滿分 100 分。

(20) 1. Given the signals $x_1(t)$ and $x_2(t)$ as follows.

(5) (a) Rewrite $x_1(t) = \sum_{n=-\infty}^{\infty} \Pi\left(\frac{t-T_0}{T_0}\right)\delta(t-nT_0)$ in the simplest form.

(5) (b) Find $X_1(f)$, the Fourier transform of $x_1(t)$.

(5) (c) Rewrite $x_1(t) = \sum_{n=-\infty}^{\infty}\left[\Pi\left(\frac{t}{T_0}\right) * \delta(t-nT_0)\right]$ in the simplest form,

where $*$ represents the convolution operation.

(5) (d) Find $X_2(f)$, the Fourier transform of $x_2(t)$.

2. (A) Consider an audio signal with spectral components in the range 300 to 3000 Hz. Assume that a sampling rate of 7kHz is used to generate a PCM signal. (a) For S/N=30 dB, what is the number of uniform quantization levels needed? (b) What data rate is required? (10%)

(B) Ten analog signals that are bandlimited to frequencies below 16 kHz are samples at the Nyquist rate. The PCM digitizing error is below 0.2%. The signals are carried by a TDM channel. What is the data rate required for the channel? (10%)

(20%) 3. $G_1, G_2, X_1, X_2, X_3, X_4$ are independent Gaussian random variables with zero means, $E[G_1] = E[G_2] = E[X_1] = E[X_2] = E[X_3] = E[X_4] = 0$. Their variances are given by var G_1 = var $G_2 = \sigma^2$ and var X_1 = var X_2 = var X_3 = var $X_4 = N_0$, where N_0 is a constant. Let $R_1 = \sqrt{(G_1 + X_1)^2 + (G_2 + X_2)^2}$ and $R_2 = \sqrt{X_3^2 + X_4^2}$.

(10%) (a). Find the probability density function $f_{R_1}(r_1)$ of R_1.

(10%) (b). Find the probability that $R_1 < R_2$.

20% 4. Consider 16-ary QAM with signals given as

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}}\left[a_i \cos w_c t + b_i \sin w_c t\right] \quad 0 \le t \le T_s,$$

where $a_i, b_i \in \{\pm 1, \pm 3\}$ with equal probability. Under the assumption of the AWGN channel with double-sided power spectral density $N_0/2$.

5% (a) Devise an optimal coherent detector for 16-QAM;

5% (b) Show the signal constellation and the optimal decision regions;

10% (c) Compute the detected error probability in term of E_s/N_0.

5.(20%)Consider a systematic block code whose parity-check equations are

$c_1 = m_1 + m_2 + m_3$

$c_2 = m_2 + m_3 + m_4$

$c_3 = m_1 + m_2 + m_4$

$c_4 = m_1 + m_3 + m_4$

where m_i are message digits and c_i are check digits.

(A)Find the generator matrix

(B)Find the parity-check matrix

(C)Encode the message (1101)

(D)Is the vector 1101001 a codeword?

6 (20%) For a relay-satellite-to-user link :

(a). Relay satellite input power: 10 W

(b). Relay satellite antenna gain: 38 dB

(c). Transmit frequency: 10 GHz

(d). Receiver noise temperature of user (antenna included): 1000K

(e). User satellite antenna gain: 15 dB

(f). Total system losses: 3 dB

(g). System bandwidth: 3000 Hz

(h). Relay-user separation: 41000 km

Knowing that $10\log_{10}(kT_0) = -174$ dBm/Hz, where k is Boltzmann's constant and T_0 is room temperature 290 K, find the signal-to-noise power ratio in a 3000 Hz bandwidth at user satellite receiver output.