

所別： 資訊工程學系碩士班 不分組科目： 離散數學與線性代數

※ 請務必按照題號次序寫在答案紙上。

1. (25%)
- (10%) (a) Define  $f: Z \rightarrow Z$  by  $f(x) = 3x^3 - x$ . Determine whether or not  $f$  is one-to-one and/or onto.
- (15%) (b) Let  $a$  and  $b$  be real numbers with  $a < b$ . Show that the set  $R^+$  of positive real numbers has the same cardinality as the open interval  $(a, b) = \{x \in R \mid a < x < b\}$ .
2. (25%) Let  $A = \{1, 2, 3, 5, 6, 9, 10, 25, 27\}$  and, for  $a, b \in A$ , define  $a \preceq b$  if and only if  $\frac{a}{b}$  is an integer.
- (7%) (a) Prove that  $\preceq$  defines a partial order on  $A$ .
- (7%) (b) Draw the Hasse Diagram for  $\preceq$ .
- (6%) (c) List all minimum, minimal, maximum and maximal elements.
- (5%) (d) Is  $(A, \preceq)$  totally ordered? Explain.
3. (10%) Which ones of the following statements are or are not logical equivalent to " $Ax = b$  has solution."
- (a)  $b$  is a linear combination of the columns of  $A$ .
- (b) Augmented matrix  $[Ab]$  has a pivot position in every row.
- (c)  $A$  has linearly independent columns.
- (d)  $Ax = b$  has free variable.
- (e)  $Ax = 0$  has only trivial solution.
- (每小題答對給 2 分，答錯扣 2 分，不答 0 分，本題總分  $\geq 0$ )
4. (10%) Find  $\lim_{k \rightarrow \infty} \begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix}^k$ .
5. (10%) You must have detailed explanation.
- (5%) (a) Which of rotation, translation, scaling, and perspective projection are linear transformation?
- (5%) (b) What formulas will make the non-linear transformations into matrix transformations?
6. (5%) (a) Show that all elementary matrices are invertible.
- (5%) (b) Let  $T$  be a linear transformation. Show that if  $\{T(v_1), T(v_2), \dots, T(v_p)\}$  is linearly independent, then  $\{v_1, v_2, \dots, v_p\}$  is linear independent.
7. (2%) (a) When will you find the least-squares solution for  $Ax = b$ ?
- (2%) (b) What is the least-squares solution for  $Ax = b$ ?
- (2%) (c) In what situation, you will compute a least-squares solution through a  $QR$  factorization?
- (2%) (d) In what condition, you can compute a least-squares solution through a  $QR$  factorization?
- (2%) (e) What is the  $QR$  factorized least-squares solution for  $Ax = b$ ?

**GOOD LUCK !**

參考用