系所別：資訊工程學系　　科目：線性代數

※ 請務必按照題號次序寫在答案紙上。

1. (10%) Give the condition(s) for that matrix A
   (a) is invertible (can't use determinant).
   (b) has zero eigenvalue.
   (c) is diagonalizable.
   (d) can be UL factorized.
   (e) can be QR factorized.
   (每小題答對給 2 分，答錯扣 2 分，不答 0 分，本題總分≥ 0)

2. (10%) Determine if each vector set is a subspace or not, respectively:
   (a) $\begin{bmatrix} 3a+b \\ 4 \\ a-5b \\ a-b \end{bmatrix}$, (b) $\begin{bmatrix} a-b \\ b-a \\ c-a \\ b \end{bmatrix}$, (c) $\begin{bmatrix} a \\ -b \\ c \\ -d \end{bmatrix}$, (d) $\begin{bmatrix} -a+1 \\ a-6b \\ 2b+a \\ 6b-a \end{bmatrix}$, (e) $\begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix}$.
   (每小題答對給 2 分，答錯扣 2 分，不答 0 分，本題總分≥ 0)

3. (10%) Describe the necessary condition(s) for a linear system $\boldsymbol{Ax} = \boldsymbol{b}$ to be consistent.

4. (10%) Give method(s) to test whether a vector set is linearly independent or not.
   (是方法不是定義)

5. (10%) (a) (2%) Give the definition of linear transformation.
   (b) (8%) Show that every linear transformation is a matrix transformation.

6. (10%) A is a square matrix. If A is invertible, show that A^T is invertible.

7. (10%) Explain the meaning of the following sentence "A basis of a vector space is a maximal independent set and a minimal spanning set."

8. (10%) Let T be a one-to-one linear transformation. Show that if vector set $\{T(\boldsymbol{v}_1), \ldots, T(\boldsymbol{v}_n)\}$ is linearly dependent, then vector set $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ is linearly dependent.

9. (10%) (a) (8%) Find a real general solution of linear system $\begin{cases} y'_1 = -y_2 \\ y'_2 = 2y_1 \end{cases}$.
   (b) (2%) Describe its types of solution trajectory: nodes, saddle points, centers, or spiral points.

10. (10%) Derive the least squares solution $\hat{\boldsymbol{x}} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}$ for the inconsistent system $\boldsymbol{Ax} = \boldsymbol{b}$.