國立中央大學九十一學年度碩士班研究生入學試題卷

所別：資訊工程學系 不分組　　科目：離散數學　　共 1 頁 第 1 頁

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1. (25 Points) Apply the Eulcidean algorithm to find the multiplicative inverse of 89(mod 233).

2. (25 Points) Let $X = \{1,2,3,4,5\}$, $Y = \{3,4\}$, and $C = \{1,3\}$. Define the relation R on $P(X)$, the set of all subsets of X, as

$$ARB \text{ if } A \cup Y = B \cup Y$$

   (a) Show that R is an equivalence relation.
   (b) List the elements of $[C]$, the equivalence class containing C.
   (c) How many distinct equivalence classes are there?

3. (25 Points) (a) Let a_n be the number of ways of forming a line of n people distinguished only by sex. For example, there are four possible lines of two people – MM, MW, WM, WW – so $a_2 = 4$. Find a recurrence relation satisfied by a_n and identify the sequence $a_1, a_2, a_3, \ldots$.

   (b) Let a_n be the number of ways in which a line of n people can be formed such that no two males are standing beside each other. For example, $a_3 = 5$ because there are five ways to form lines of three people with no two males beside each other; namely, FFF, MFF, FMF, FFM, MFM. Find a recurrence relation satisfied by a and identify the sequence $a_1, a_2, a_3, \ldots$

4. (25 Points) Suppose Carling has 11 weeks to prepare for her tournament, that she intends to play at least one set a day and at most 132 practice sets in all. Show that during some period of consecutive days, Carling will play precisely 21 sets.