國立中央大學八十九學年度碩士班研究生入學試題卷

125 所別：資訊工程學系 不分組 科目：線性代數 共 1 頁 第 1 頁

※ 請務必按照題號次序寫在答案紙上。

1.(15%) A *Givens rotation* is a linear transformation from R^n to R^n used in computer programs to create zeros in a vector. The standard matrix of a *Givens rotation* in R^2 has the form: $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, $a^2+b^2=1$.

(a) Find a and b such that vector $[4, 3]^T$ is rotated into $[5, 0]^T$. (8%)

(b) Find a 3×3 matrix A such that $A\,[2, 3, 4]^T = [\sqrt{29}, 0, 0]^T$. (7%)

(Hint: Find a Givens rotation in $\mathbf{R}^3$ s.t. $\begin{bmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & a \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2\sqrt{5} \\ 3 \\ 0 \end{bmatrix}$. Then apply another Givens rotation in $\mathbf{R}^3$)

2.(10%) Let $A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 5 \\ 1 & 5 \\ 3 & 4 \end{bmatrix}$. Compute $A^{-1}B$ without computing A^{-1}.

3.(15%) A polynomial $p(t)$ of degree n-1 is defined as $p(t) = c_0 + c_1 t + c_2 t^2 + \cdots + c_{n-1} t^{n-1}$, where $c_0, c_1, c_2, \ldots, c_{n-1}$ are n real numbers. Given n arbitrary real numbers $y_1, y_2, \ldots, y_n$ and n distinct real numbers $x_1, x_2, \ldots, x_n$, show that there exists one and only one polynomial $p(t)$ of degree n-1 such that $p(x_1) = y_1, p(x_2) = y_2, \ldots, p(x_n) = y_n$.

4.(10%) Compute the determinant $\begin{vmatrix} 4 & 8 & 8 & 8 & -3 \\ 0 & 1 & 0 & 0 & -1 \\ 6 & 8 & 8 & 8 & -1 \\ 0 & 8 & 8 & 3 & -8 \\ 0 & 8 & 2 & 1 & -7 \end{vmatrix}$.

5.(10%) True or false for determinants (每小題答對給 2 分，答錯扣 2 分，不答 0 分)

(a) $\det AB = \det A \det B$

(b) $\det(A+B) = \det A + \det B$

(c) $\det A^T = \det A$

(d) $\det(rA) = r \det A$

(e) $\det A = \det B$ if B is produced by interchanging two rows of A.

6.(10%) True or false for eigenvalues (每小題答對給 2 分，答錯扣 2 分，不答 0 分)

(a) If λ is an eigenvalue of A, then λ^{-1} is an eigenvalue of A^{-1}.

(b) A and A^T have the same eigenvalues.

(c) If A^2 is a zero matrix, then 0 is the only eigenvalue of A.

(d) A is invertible if and only if 0 is not an eigenvalue of A.

(e) A is diagonalizable if and only if all eigenvalues of A are different (distinct).

7.(10%) Let W be a subspace of R^n and let $W^\perp$ be the orthogonal complement of W. Show that $W^\perp$ is a subspace of R^n.

8.(10%) (a) Find a spanning set for the null space of matrix $\begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$. (5%)

(b) Explain why the spanning set is automatically linearly independent. (5%)

9.(10%) Find a singular value decomposition of matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$.

GOOD LUCK!