國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 資訊工程研究所 不分組 科目: 線性代數 共 1 頁 第 1 頁

※ 請務必按照題號次序寫在答案紙上。

1. (40 %) True and False. (一定要有說明或反例)
   (a) Two linear systems $Ax = b$ and $Bx = c$ are equivalent then A and B are row equivalent.
   (b) If a linear system has no free variables, then it has a unique solution.
   (c) A is a square matrix. If linear transformation $x \mapsto Ax$ is onto, then $x \mapsto Ax$ is one-to-one.
   (d) Let T be a linear transformation from R^3 to R^m. If vectors a, b, c are linearly independent, then $T(a)$, $T(b)$, $T(c)$ are linearly independent.
   (e) The nonempty subset of a linear-dependent vector set is linearly dependent.
   (f) $n \times m$ matrix A has n distinct eigenvalues if and only if A is diagonalizable.
   (g) If matrix A is diagonalizable, then the columns of A are linearly independent.
   (h) If $n \times n$ matrix A has n linear-independent eigenvectors, then so do both A^T and A^{-1}.

2. (10 %) Find the c_1, c_2, and c_3 in the equation $c_1\begin{bmatrix}0\\1\\-3\end{bmatrix}+c_2\begin{bmatrix}5\\2\\-1\end{bmatrix}+c_3\begin{bmatrix}3\\1\\2\end{bmatrix}=\begin{bmatrix}-1\\0\\1\end{bmatrix}$.

3. (10 %) Explain why the linear transformation $T: R^n \to R^m$
   (a) is onto, then $n \geq m$.
   (b) is one-to-one, then $n \leq m$.

4. (10 %) Find a matrix A such that the transformation $x \mapsto Ax$ takes $\begin{bmatrix}1\\3\end{bmatrix}$ and $\begin{bmatrix}2\\7\end{bmatrix}$ into $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}3\\1\end{bmatrix}$, respectively.

5. (10 %) Let A and B be $n \times n$ matrix. Which one of the two statements: (i) $\det AB = \det A \det B$ and (ii) $\det(A+B) = \det A + \det B$ is wrong ? What conditions on the matrices and matrix addition make the wrong statement to be right ? Note that the "det" is determinant.

6. (10 %) Find bases for Row A, Col A, and Nul A, where $A = \begin{bmatrix}1&1&3&3&1\\2&3&7&8&2\\2&3&7&8&3\\3&1&7&5&4\end{bmatrix}$.

7. (10 %) Find a QR factorization of matrix $\begin{bmatrix}1&3&5\\-1&-3&1\\0&2&3\\1&5&2\\1&5&8\end{bmatrix}$, where columns of Q form an orthonormal basis for Col A.