

國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 資訊工程研究所 不分組 科目:

離散數學

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1. You are visiting an island containing two types of people: **knights** who always tell the truth and **knaves** who always lie.

(a). Two natives **A** and **B** speak to you as follows:

A says: **B** is a knight.

B says: **A** and I are of opposite type.

What are **A** and **B**?(5%)

(b). Another two natives **C** and **D** approach you but only **C** speaks.

C says: Both of us are knaves.

What are **C** and **D**?(5%)

(c) You then encounter natives **E** and **F**.

E says: **F** is a knight.

F says: **E** is a knight.

What are **E** and **F**?(5%)

(d) Finally you meet a group of six natives, **U**, **V**, **W**, **X**, **Y**, and **Z**, who speak to you as follows:

U says: None of us is a knight.

V says: At least three of us are knights.

W says: At most three of us are knights.

X says: Exactly five of us are knights.

Y says: Exactly two of us are knights.

Z says: Exactly one of us is a knight.

Which are knights and which are knaves.(10%)

2. (a) Does there exist a binary relation **R** on $\{1,2,3\}$ such that **R** is reflexive, transitive, symmetric, and antisymmetric? Justify your answer.(7%)

(b) Is the union of two equivalence relations always an equivalence relation? Justify your answer.(8%)

(c) If **a**, **b**, and **c** are odd integers, can $ax^2 + bx + c = 0$ have a rational solution? Justify your answer.(10%)

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3. a) Use an example to briefly describe Dijkstra's algorithm for finding a shortest path between two given vertices of a graph. (15%)
- b) Consider the condition that the costs of some edges of the input graph are allowed to be negative. Does Dijkstra's algorithm still work under this condition? Justify your answer. (10%)
4. Compute the numbers of integer solutions for the following equations and inequalities with constraint. (25%)
- a) $X_1 + X_2 + X_3 = 20$ where X_1, X_2, X_3 are nonnegative integers
- b) $X_1 + X_2 + X_3 < 20$ where X_1, X_2, X_3 are nonnegative integers
- c) $2X_1 + X_2 + X_3 = 20$ where X_1, X_2, X_3 are nonnegative integers
- d) $X_1 + X_2 + X_3 \leq 20$ where $1 \leq X_1, X_2 \leq 10$ and $1 \leq X_3 \leq 7$.