國立中央大學113學年度碩士班考試入學試題

所別： 資工類

科目： 離散數學與線性代數

＊本科考試禁用計算器

第一部分: 共 50分，單選題，每題五分，錯一題倒扣2分，
扣到單選題[整大題]0分為止

1. Let $A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 4 & -1 & 9 \end{bmatrix}$, and the LU decomposition of A be

   $\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$. What is $\lfloor a+b+c+d+e+f+g+h+i \rfloor \%5$?

   (% is the modulo operation. $\lfloor z \rfloor$ rounds z to the smaller nearest integer.)

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

2. Suppose that in P_3 we want to change from the ordered basis $[1, x, x^2]$ to the ordered basis $[1, 2x, 4x^2 - 2]$. Let the transition matrix from the first basis to the second basis be S, and the number of zeros in S be D. What is D%5?
   (% is the modulo operation.)

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

3. Let A be a 4×4 matrix with reduced row echelon form given by $U = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Let the first two columns of A be $\begin{bmatrix} 3 \\ 2 \\ -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ -2 \\ 3 \end{bmatrix}$, and denote the third column of A as $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$. What is $\lfloor a+b+c+d \rfloor \%5$?

   (% is the modulo operation. $\lfloor z \rfloor$ rounds z to the smaller nearest integer.)

注意：背面有試題

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(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

4. Let the matrix $A = (a_{ij})$ represent the composite transformations "a yaw of 45°, followed by a pitch of −90° and then a roll of −45°". What is $Round\{|\sum a_{ij}|\}\%5$? (% is the modulo operation. Round{z} rounds z to the nearest integer.)
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

5. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. What is the dimension spanned by the eigenvectors of A?
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

6. $\boldsymbol{A} = \begin{bmatrix} 5 & 2 & 2 \\ 3 & 6 & 3 \\ 6 & 6 & 9 \end{bmatrix}$, $\boldsymbol{B} = \mathbf{A}^3 - 20\mathbf{A}^2 + 92\boldsymbol{A} - 120\boldsymbol{I}_{3x3}$. D is the determinant of $\boldsymbol{B}$. What is $|D|\%5$? (% is the modulo operation. $|.|$ is the absolute value.)
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

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7. The subspace U of $\mathbf{R}^4$ is spanned by the three vectors:
   $\mathbf{v}_1=[1,-1,-1,1]^T$, $\mathbf{v}_2=[1,2,-3,2]^T$, $\mathbf{v}_3=[3,3,0,-2]^T$.
   Use the Gram -Schmidt process to find the orthonormal basis of U:
   $\mathbf{t}_1=[\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2}]^T$, $\mathbf{t}_2=[m,n,p,q]^T$, $\mathbf{t}_3=[r,s,x,y]^T$. $D=\left(\text{Round}\left\{\frac{1}{n^2s^2}\right\}\right)\%5$. What is D?
   (% is the modulo operation. Round{z} rounds z to the nearest integer.)
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

8. Following the previous question. The subspace $V=U^{\perp}$ is U's orthogonal complement in $\mathbf{R}^4$. Given a vector w=[10,0,8,2]^T, find $w=v+u$, where $v=[a,b,c,d]^T\in V, u=[e,f,g,h]^T\in U$. What is $(\text{Round}\{c^2+g^2\})\%5$?
   (% is the modulo operation. Round{z} rounds z to the nearest integer.)
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

9. $\boldsymbol{A}=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$. $\boldsymbol{P}$ is used to diagonalize $\boldsymbol{A}$ by $\boldsymbol{P}^{-1}\boldsymbol{AP}=\boldsymbol{D}$, where
   $\boldsymbol{D}=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$, $a\le b\le c$, and $\boldsymbol{P}=\begin{bmatrix}d & e & 1\\ -1 & f & g\\ h & 1 & j\end{bmatrix}$. What is the value K,
   $K=(\text{Round}\{|a+b+c+d+e+f+g+h+j|\})\%5$?
   (|.| is the absolute value. % is the modulo operation. Round{z} rounds z to the nearest integer.)
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

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10. We are required to find the parabola $C + Dt + Et^2$ that comes closest to the values **v**=(0,2,2,5) at the times $\boldsymbol{t}$=(0,1,3,4). What is $F = \text{Round}\{|C + D + E| \times 256\}\%5$? (|.| is the absolute value. % is the modulo operation. Round{z} rounds z to the nearest integer.)
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

第二部分: 共 50 分，多選題，每題 5 分，每個選項單獨計分，每個選項 1 分，答錯一個選項倒扣 1 分，扣到多選題[整大題]0 分為止。

11. Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph?
(a) K_5
(b) K_6
(c) $K_{3,3}$
(d) $K_{3,4}$
(e) $K_{4,4}$

12. Draw a graph with 64 vertices representing the squares of a chessboard. Connect two vertices with an edge if you can move legally between the corresponding squares with a single move of a knight. [The moves of a knight are L-shaped, two squares vertically (or horizontally) followed by one square horizontally (respectively, vertically).]
(a) This graph is bipartite.
(b) The largest degree number of the graph is 10.
(c) The smallest degree number of the graph is 4.
(d) There are four vertices of degree 2.
(e) There are eight vertices of degree 3.

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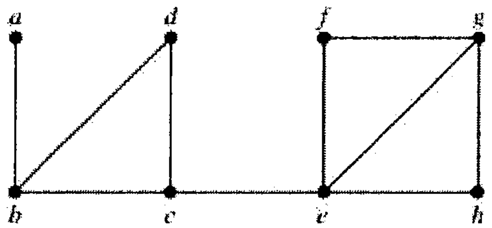
13. A sequence $d_1, d_2, \ldots, d_n$ is called graphic if it is the degree sequence of a simple graph. Which of these sequences are graphic?
    (a) 5, 4, 3, 2, 1, 0
    (b) 6, 5, 4, 3, 2, 1
    (c) 2, 2, 2, 2, 2, 2
    (d) 3, 3, 3, 2, 2, 2
    (e) 3, 3, 2, 2, 2, 2

14. Find the cut vertices and cut edges in the following graph.
    (a) The cut vertices are b, c, d and e.
    (b) The cut vertices are b, c, and e.
    (c) The only cut edge is {c, e}.
    (d) The cut edges are {a, b} and {c, e}.
    (e) The cut edges are {b, d} and {c, e}.



15. Let G be a graph with at least three vertices.
    (a) If there is a Hamiltonian path between any two vertices of G, G must contain a Hamiltonian cycle.
    (b) If, at every vertex v in G, there is a Hamiltonian path which starts at v, G must contain a Hamiltonian cycle.
    (c) If there is an Eulerian trail (path) between any two vertices of G, G must contain an Eulerian circuit.
    (d) $K_{m,n}$ is Hamiltonian (a graph with a Hamiltonian cycle) if and only if $n > 1$ and $m = n$.
    (e) $K_{m,n}$ is Eulerian (a graph with a Eulerian circuit) if and only if m and n are both odd.

16. Let "P→Q is true" be a relation **R**(P,Q) between proposition P and Q, where "→ " is the **implication** logic connective. Which of the followings are true?
   (a) **R** is reflexive.
   (b) **R** is anti-symmetric
   (c) **R** is transitive
   (d) **R** is a partial ordering relation.
   (e) The transitive closure of **R** is **R** itself

17. Suppose a set S, ｜S｜=3. Consider possible function $f: S \times S \rightarrow S$. Which of the following statements are true?
   (a) number of possible f is 3^6
   (b) number of possible f is 3^9
   (c) number of 1-to-1 f is 3!
   (d) number of onto f is $3^9 - (3 \bullet 2^9) + 3$
   (e) number of bijection f is 1.

18. Consider Josephus Problem, where *n* people are placed clockwise around a circle, from 1, 2,..., to *n, n* is next to 1; every second person is eliminated each step (so the first few eliminated are 2,4,6,...), until the last one, say *k*, is remained, we denote *J*(*n*)=*k*. $\forall n > 1, n \in N$, Which of the followings are true?
   (a) $J(n+1) = \frac{1}{2}J(n) + 1$
   (b) $J(2n) = 2J(n) - 1$
   (c) $J(2n+1) = 2J(n) - 1$
   (d) $J(2n) = 2J(n) + 1$
   (e) $J(2n+1) = 2J(n) + 1$

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19. Which of the following statements are true?
   (a) To find greatest common divisor for 2 large integers, Euclidean algorithm provides best complexity if the 2 numbers have been prime factorized.
   (b) In a weighted graph, finding a minimum-weight Hamilton path is un-computable.
   (c) Lattice is a special case of partial ordering set.
   (d)Bipartite graph can be used to represent any function.
   (e)We can use unlimited number of "and", "or", and "not" connectives to describe the logic behavior of any logic operator.

20. Solving recurrence equation $\forall n \geq 2, 2a_n = 7a_{n-1} - 6a_{n-2}, a_0 = 3/2, a_1 = 11/4$.
   Which of the following statement are true?
   (a) The characteristic polynomial for this equation is $6x^2 - 7x + 2 = 0$.
   (b) using generating function, we have $2G(z) - 7zG(z) + 6z^2G(z) = 0$.
   (c) using generating function, we have $G(z) = \dfrac{3-5z}{2-7z+6z^2}$
   (d) $a_n = ((1/2) \bullet (3/2)^n) + 2^n$
   (e) $a_n = (1/2)^{n+1} + 2^n$