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國立中央大學94學年度碩士班考試入學試題卷 共 1 頁 第 1 頁  
所別：電機工程學系碩士班 甲乙丙丁組 科目：工程數學

1. Solve the following differential equation (10%)

$$y'' - 2y' + y = 35x^{3/2}e^x + x^2$$

2. Find the eigenvalue and eigenfunction of the following Sturm-Liouville problem (10%)

$$(e^{2x}y')' + e^{2x}(\lambda + 1)y = 0, y(0) = 0, y(\pi) = 0$$

3. Solve the following integral equation (10%)

$$y(t) = \sin(2t) + \int_0^t y(\tau) \sin(2(t-\tau)) d\tau$$

4. Consider the matrix

$$A = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

- (a) Show that it is unitary. (5%)

- (b) Show that its eigenvalues are  $e^{i\theta}$  and  $e^{-i\theta}$ . (5%)

- (c) Find the corresponding eigenvectors; show that they are orthogonal. (5%)

- (d) Verify that  $U^*AU = (\text{diagonal matrix})$ , where  $U$  is the matrix of eigenvectors of  $A$ . (5%)

5. Calculate

$$\lim_{N \rightarrow \infty} \left( I - i \frac{\theta}{N} \sigma \right)^N, \text{ where } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \sigma = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}. (10\%)$$

6. Show that the limit of the function

$$f(z) = \left( \frac{z}{\bar{z}} \right)^2$$

as  $z$  tends to 0 does not exist, where  $z$  is complex number and  $\bar{z}$  is its complex conjugate. (10%)

7. Derive the Laurent series representation for the function

$$f(z) = e^{\frac{1}{z}} \quad (0 < |z| < \infty). (10\%)$$

8. Use residues to evaluate the improper integral

$$\int_0^{\infty} \frac{x^2}{x^6 + 1} dx. (10\%)$$

9. Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 1 & \text{if } |x| < 1, \\ 0 & \text{if } |x| > 1. \end{cases} (10\%)$$