國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 電機工程研究所 甲、乙、丙、丁組 科目: 工程數學 共 2 頁 第 1 頁

1. (12pt, 3 pt each) Integrate

$$f(z) = \frac{2z^2 + 2}{z^2 - 1}$$

in the counterclockwise sense around a circle of radius 1 with center at the point

(a) $z = 1$ (b) $z = \frac{1}{2}$ (c) $z = -1 + \frac{1}{2}i$ (d) $z = i$

with i being the imaginary unit, i.e., $i = \sqrt{-1}$.

2. (10pt) Consider rolling three fair dice (of the same size) at a time in a casino. The game is to bet on the numbers that the three dice show up. Let n_1, n_2, and n_3 be the three numbers that show up in a single roll.

   (a) (3pt) Find the probability of the event that $n_1 = n_2 = n_3$.

   (b) (4pt) Find the probability of the event that $n_1 < n_2 < n_3$.

   (c) (3pt) Find the probability of the event that $n_1 + n_2 + n_3 = 12$.

3. (8pt, 4pt each) Using the residue integration method, derive the following real integrals:

$$\text{(a)} \quad \int_0^\infty \frac{2}{1+x^4}dx \qquad \text{(b)} \quad \int_{-\infty}^\infty \frac{x^2-1}{x^4+3x^2+2}dx.$$

參考用

4. Solve $\mathbf{X}' = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\mathbf{X}$. (10%)

5. Solve $\mathbf{X}' = \begin{bmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}\mathbf{X}$. (10%)

6. Solve the system $\mathbf{X}' = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}\mathbf{X} + \begin{bmatrix} -8 \\ 3 \end{bmatrix}$ on $(-\infty, \infty)$. (15%)

(7) (15%) For a second order differential equation

$$y'' + p(x)y' + q(x)y = r(x), \tag{1}$$

with arbitrary variable functions $p(x)$, $q(x)$ and $r(x)$ those are continuous on some interval I. If the linearly independent solutions of the homogenous equation

$$y'' + p(x)y' + q(x)y = 0, \tag{2}$$

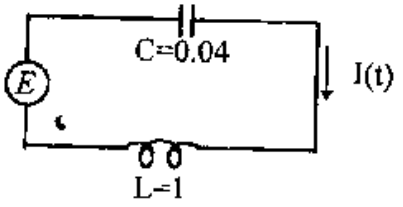
are y_1 and y_2 and define that $W = y_1y_2' - y_2y_1'$ is the Wronskian of y_1 and y_2. Please prove that the particular solution y_p of (1) on I can be obtained by

$$y_p = -y_1 \int \frac{y_2 r(x)}{W} dx + y_2 \int \frac{y_1 r(x)}{W} dx.$$

(8) (10%) Consider the LC circuit given in the following figure with $I(0) = I'(0) = 0$. The voltage is given by

$$E(t) = \begin{cases} 25t, & 0 \le t \le 4, \\ 100, & t > 4. \end{cases}$$

Find the current $I(t)$ for all values of $t \ge 0$.



(9) (10%) Solve the integral equation

$$y(t) = t + \int_0^t y(\tau) sin(t - \tau) d\tau.$$