tive-velocity is constant during the expansion; this is also true for V_1 and V_2 separately. This prediction follows because the dipole field configuration does not have a characteristic radius; thus c is the only dimensional quantity in the problem. With the simplest assumptions, it is also predicted that V/c will be the same for different bursts. In the case of 3C120, the velocities of the two bursts observed may be different (although the analysis of the second burst is only preliminary[1]). This could indicate that particles of the second burst move on field lines distorted by the first burst (as the distortion preserves the axial symmetry, PA is still expected to be the same) or that the analysis of the observations of the second burst has not properly included the contribution of the first.

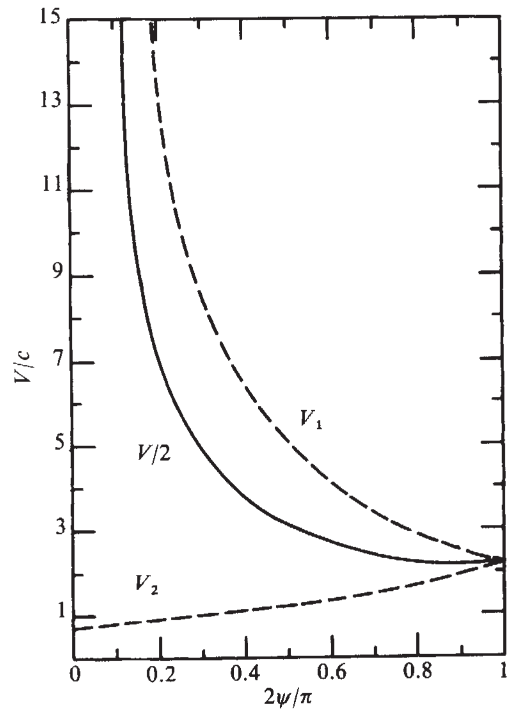
The prediction $V/c \geqslant V_m c = 4.4$ means that the apparent relative velocity is always superluminal with a minimum possible value of $4.4c$ which occurs when the angle ψ between the line of sight and dipole axis is $\pi/2$. This result can be used to impose constraints on the Hubble constant, H_0, and the deceleration parameter, q_0, which enter into the determination of V from the observed angular velocity of separation. For example, H_0 must be $<53\ \mathrm{km\,s^{-1}Mpc^{-1}}$ in order to make the inferred value of V/c for 3C273 larger than 4.4 for $q_0 = 0$ and $H < 49\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ for $q_0 = 1$. Also $H_0 < 63\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ in order to make V/c larger than 4.4 in the first burst from 3C120. Thus, if the dipole-field model is correct, $H_0 \leqslant 55\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. When enough superluminal sources are observed H_0 and q_0 could be determined.

The prediction that $d(\mathrm{PA})/d\lambda = 0$; $d(V/c)/d\lambda = 0$ simply states that both the position angle and velocity are independent of wavelength (which is different from what would be expected on the basis of explanations based on propagation and opacity effects).

The relationships $V_1 = V_1(V)$; $V_2 = V_2(V)$, imply that the individual velocities V_1 and V_2 are specified functions of the relative velocity V (or, equivalently, the angle ψ). The numerical relationship is given in Fig. 2. This prediction should be testable by future observations although no measurements of individual velocities have yet been reported.

The final prediction states that the probability of finding V between V_m and some value V_0 is $\cos \psi(V_0/c)$, where $\psi(V/c)$ is given in Fig. 2. We have assumed that the dipole axes are randomly orientated in space for the sample under consideration. One of the four well-observed sources, 3C279, has an apparent superluminal velocity of $10c$ (ref. 1). One may ask if this value is too large to be considered plausible for the limited observational sample? Using the results shown in Fig. 2, we find that the probability of observing one source with $V/c \geqslant 10$ in a sample of four sources is ~ 0.4, which is acceptable.

Fig. 2 The calculated apparent velocities of the faster (V_1), and slower (V_2), components and the presently observed relative velocity (V) as functions of the angle ψ between the dipole axis and the line of sight. A pure dipole field configuration is assumed.



Sanders[3] mistakenly concluded that 4.4 is the maximum value of V/c which can be achieved. Thus, his results apparently conflict with observations for three of the four well-studied sources (3C345, 3C120, and 3C279).

To avoid this problem, Sanders assumed that particles which move on different field lines are ejected at different times. This additional assumption conflicts, in general, with the observationally verified conditions that the relative velocity is constant and that the observed rise time of the burst is short. Sanders also limited his calculations to the special case of $\psi = 90°$ (see Figs 1 and 2) and discussed only predictions (1) and (4) given above.

In principle, it is possible to see two more sources between A and B. These additional sources represent particles that are returning to the origin on the other side of the central object. However, the existence of these additional sources is uncertain as the observer can see them only after two oppositely moving beams have interacted. In any case, these backside points will appear to move with subluminal relative velocities which are always less than $0.1c$.

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MORDEHAI MILGROM

Department of Nuclear Physics,
Weizmann Institute of Science

JOHN N. BAHCALL

Department of Nuclear Physics,
Weizmann Institute of Science, Rehovot, Israel
and
Institute for Advanced Study, Princeton, New Jersey 08540

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Anomalous acceleration of minor ions in the solar wind

OBSERVATIONS of the solar wind plasma have shown that the bulk speed of helium ions often exceeds that of the protons, provided conditions are such that Coulomb collisions are ineffective in coupling the two species together[1,2]. We describe here a mechanism which could produce this unexpected effect for any minor ion species, based on the assumption that the sun is a copious emitter of Alfvén waves, as is indeed observed[3].

The (nonradial) flow of a minor ion species in a purely radial background solar wind containing a spiral interplanetary magnetic field is governed by the equation[4]:

$$u\frac{\mathrm{d}u}{\mathrm{d}r} = \left[\frac{\Omega r}{U}\left(u\frac{\mathrm{d}w}{\mathrm{d}r} + \frac{uw}{r}\right) + \frac{w^2}{r}\right] + \nu(U-u)\left[1 + \left(\frac{\Omega r}{U}\right)^2\right] + F - \frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{ZeE}{Am} - \frac{G\mu}{r^2} \qquad (1)$$

where u and w are respectively the radial and azimuthal velocity components, Ω is the angular frequency of solar rotation, U is the solar wind (proton) speed, ν is the Coulomb collision frequency between the minor species and protons[5], F is a wave force[6], G is the gravitational constant, μ is the solar mass, p and ρ are respectively the partial pressure and density of the

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minor species (assumed to have charge Ze and mass Am) and E is the radial (charge-separation) electric field associated with the electron pressure gradient. It can be shown that the latitudinal velocity component v is small and hence that the azimuthal velocity is given by $w = (U-u)\Omega r/U$ (ref. 4). The first term on the right hand side of equation (1) represents effects due to the Sun's rotation and the spiral nature of the interplanetary magnetic field (B_r, B_φ), which give rise to a centrifugal force, w^2/r, and a Lorentz force, $(Ze/Am)vB_\varphi$.

In the dense plasma near the Sun where collisions with protons are frequent, Coulomb friction accelerates minor ions outwards against the gravitational force, part of which is in any case cancelled by the charge-separation electric field. (The partial pressure gradient of the minor ions should normally have very little effect unless the ion temperature is very high[7].) Once the effects of gravity are overcome, the wave force, the electric field and the rotational force (given by the first term on the right hand side of equation (1)) assist Coulomb friction in further increasing the minor ion speed so that it becomes comparable with and perhaps even exceeds the solar wind speed. Eventually, well beyond the Alfvén critical point, we expect the acceleration to be small so that the minor ion flow is determined essentially by the first three terms on the right of equation (1) corresponding to the effects of rotation, Coulomb friction, and waves respectively. The force due to Alfvén waves is given by[4,6]

$$F = \delta\left\{((U+V)^2 - u^2)\left(\frac{1}{r} + \frac{1}{2U}\frac{dU}{dr}\right) - 2(U+V)u\frac{d}{dr}\left(\frac{u}{U+V}\right)\right\} + F_{res}$$

$$\equiv F_w + F_{res} \qquad (2)$$

where V is the Alfvén speed and $\delta = B_w^2/2B^2$ with B the magnetic field strength and B_w the wave magnetic field, determined on the basis of the WKB method[8] and F_{res} is the contribution to the wave force arising from waves whose Doppler shifted frequency matches the ion gyrofrequency. The wave force on the solar wind, F_p say, is the gradient of the wave pressure $B_w^2/2\mu_0$, which on using the WKB variation of B_w may be written

$$F_p = \delta V^2\left\{\frac{3}{r} + 2\frac{d}{dr}(\log(U+V)) - \frac{1}{2U}\frac{dU}{dr}\right\} \qquad (3)$$

Since $F_w \gtrless F_p$ according as $u \lessgtr U$ resonant wave acceleration; F_{res}, is necessary (in the absence of other effects) to achieve minor ion speeds exceeding the solar wind speed.

However, once $u > U$ has been accomplished by resonant acceleration we observe that the non-resonance wave force tends to bring the minor ion flow into an equilibrium such that $u = U + V$. On the other hand, Coulomb friction and rotational effects tend to bring the ion flow into equilibrium such that $u = U$. In general therefore, we expect the minor flow speed to be 'trapped' in the range $U \lesssim u \lesssim U + V$ and hence that the acceleration is indeed small. The 'quasi-equilibrium speed', obtained from putting $du/dr = 0$ in equation (1) in which we use equation (2) for F_w and for simplicity ignore F_{res} and neglect spatial variations of U and $U + V$, is given by,

$$u = u_e = \frac{U}{2(\xi+\delta)}\left[\left\{\eta^2(1+\xi)^2 + 4(\xi+\delta)\right.\right.$$
$$\left.\left.\times\left(\xi + \eta(1+\xi) + \delta(M^{-1}+1)^2 + \frac{rF_e}{U^2}\right)\right\}^{1/2} - \eta(1+\xi)\right] \qquad (4)$$

where

$$F_e = -\frac{1}{\rho}\frac{dp}{dr} + \frac{ZeE}{Am} - \frac{G\mu}{r^2}$$

$M \equiv U/V$, $\xi \equiv (\Omega r/U)^2$ is a measure of the strength of rotational effects and $\eta \equiv (\nu r/U)$ is the ratio of the solar wind expansion time to the ion–proton collision time. The relation

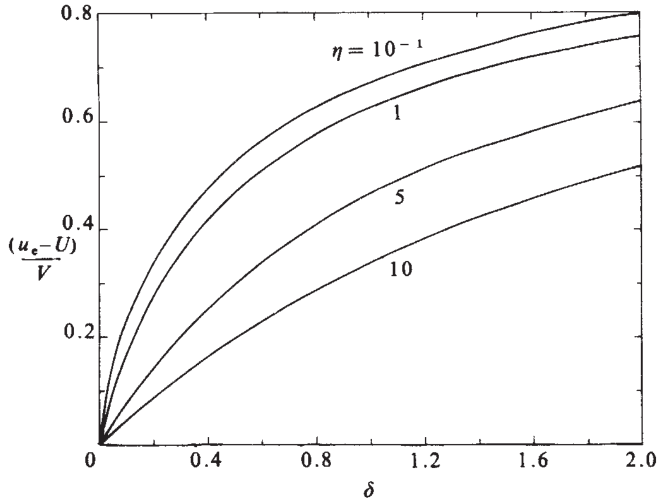

Fig. 1 Variation of the equilibrium differential minor ion speed with wave energy flux (δ) for various values of the ratio η = (solar wind expansion time/Coulomb collision time) for typical conditions at $r = 1$ AU. $\xi = 1$, $M = 8$.

(4) is shown in Fig. 1 for the case $\xi = 1$ and $M = 8$ corresponding to conditions typically observed at $r = 1$ AU for which we can neglect rFe/U^2 which is $0(1/M^2)$.

This result is in good qualitative agreement with the findings of Neugebauer[2] to the effect that the differential flow speed between helium ions and protons in the solar wind is controlled by the parameter η, as is the temperature difference. We predict on this basis that the difference in flow speeds decreases with increasing heliocentric distance (increasing ξ) and, provided there is sufficient wave energy flux and Coulomb friction is ineffective, that it increases with decreasing heliocentric distance (increasing V and decreasing ξ). This is made evident by putting $\eta = 0$ in equation (3), which yields $u_e \to U$ as $\xi/\delta \to \infty$ and $u_e \to U + V$ as $\xi/\delta \to 0$.

We stress that the 'quasi-equilibrium speed' given by equation (4) is qualitative because resonant wave acceleration has been neglected and the effects of the waves on the solar wind are not dealt with consistently.

In general ν and η are functions of $(U - u)$ and of the proton and minor ion temperatures and it is possible for more than one equilibrium speed to exist. For this to be so, it is necessary that the thermal speeds of the protons and minor ions should be small compared with V, in which case two stable equilibrium speeds may exist, one such that $u_e \approx U$ and the other such that $u_e \approx U + V$.

We are now performing numerical calculations to determine the combinations of wave energy and solar wind proton fluxes necessary to produce minor ion bulk speeds exceeding the proton bulk speed in accordance with observations and to determine the conditions under which more than one equilibrium speed can occur.

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J. F. McKenzie

Danish Space Research Institute,
Lyngby, Denmark

W.-H. Ip
W. I. Axford

Max-Planck-Institut für Aeronomie,
D-3411 Katlenburg-Lindau 3, FRG

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