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Estimates of galactic cosmic ray spectra at low energies

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Summary. A simple leaky-box model including shock acceleration, energy losses, and propagation loss is employed to calculate the energy spectra of cosmic ray protons, alpha particles, medium (C) and heavy (Fe) nuclei, and electrons between a few MeV/nucleon and 100 GeV/nucleon. On the basis of these theoretical results, the energy densities and pressures of the cosmic ray particle species are computed. One result of relevance to cosmic ray interaction with interstellar clouds is that the energy spectrum of protons has been found to be very flat for the energy range between 1 MeV and 100 MeV. This is in agreement with the 21-cm line observations of the optical thickness of H I in molecular clouds. Another consequence of the low energy proton and electron spectra so estimated is simply that the total cosmic ray pressure should be on the order of 0.4 eV cm^{-3} which implies that the low energy cosmic ray particles do not contribute significantly to the pressure balance at the boundary of the heliosphere.

Key words: cosmic rays – energy spectra – acceleration – energy loss

The energy spectra of galactic cosmic ray protons, alpha-particles and electrons below about 1 GeV/nucleon are of particular interest since protons and alpha particles provide most of the cosmic ray pressure in the interstellar medium, while low energy electrons are responsible for most of the diffuse radio emission. Unfortunately, particles in this energy range are strongly modulated by the solar wind and the energy spectra are difficult to determine in the inner solar system by direct measurement. It is not known therefore, whether the contribution of these low energy particles to the cosmic ray pressure is significant and consequently whether they play a role in determining the extent of stellar wind regions (e.g. Axford, 1972) and driving a galactic wind (e.g. Johnson and Axford, 1971). Furthermore, a large flux of low energy cosmic rays could be important in determining the ionization and energy balance in the interstellar medium as a consequence of Coulomb and other collisional losses (e.g. Field et al., 1969).

A reasonable approach to estimating low energy cosmic ray spectra theoretically is to assume that the injection of galactic cosmic ray particles by supernova shock acceleration is balanced by the energy losses due to interactions with the interstellar medium and escape from the galaxy. In this report we describe the results of such calculations using a simple leaky-box for cosmic ray propagation with an escape law which roughly approximates the effects of both convection by a galactic wind and diffusion.

Several lines of argument have suggested that cosmic ray injection is the result of shock acceleration associated with supernova explosions (Axford et al., 1977; Krimsky, 1977; Bell, 1978a, b; Blandford and Ostriker, 1978; Blandford and Ostriker, 1980; Axford, 1981). Firstly, the resultant injection spectra in momentum space have a power law of the form $F(p) \propto p^{-q}$ with q being approximately constant over a wide energy range. With reasonable assumptions concerning the propagation of supernova shocks and for the cosmic ray diffusion coefficient in the interstellar medium, the spectral index is found to be $q=4+\delta$, where $0<\delta \leqq 0.3$. More detailed calculations by Bogdan and Völk (1983) and Moraal and Axford (1983) using an "onion-shell" model of supernova acceleration have confirmed this power law nature of the injection spectra as first noted by Blandford and Ostriker (1980). Furthermore, a comparison between the energy spectra of (primary) cosmic ray protons and those of secondary particles produced by fragmentation of heavier elements suggests that diffusive escape from the galaxy takes place with a time-scale proportional to $p^{-\mu}$ when $\mu=0.5$–0.7 (Koch et al., 1981; Simon and Mathis, 1983; Ormes and Protheroe, 1983). Convective losses associated with a galactic wind are apparently important only at energies less than about 2 GeV/nucleon (Jokipii and Higdon, 1979).

Under steady-state conditions, the Boltzmann equation for cosmic ray particles with source and sink terms can be written as

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\frac{\partial p}{\partial t}f\right)=Q(p)-\frac{f}{t_e}, \tag{1}$$

where $Q(p) \propto p^{-q}$ is the source function, and t_e is the time scale for leaky loss of the cosmic ray particles. This is simpler than the equation used by Blandford and Ostriker (1980) in that we neglect entirely the effects on the cosmic ray spectra of re-acceleration. It is clear that this is a valid approach since the spectra of secondary particles are steeper than those of primaries, which could not be the case if re-acceleration in an ensemble of supernova shocks were important. In fact re-acceleration could be important in determining the effective source strength (which is arbitrary in the present work) without affecting the shape of spectra achieved by shock acceleration. This implies that the most important cosmic ray sources are relatively small, high Mach number supernova remnants (e.g. Axford, 1981; Bogdan and Völk, 1983; Moraal and Axford, 1983).

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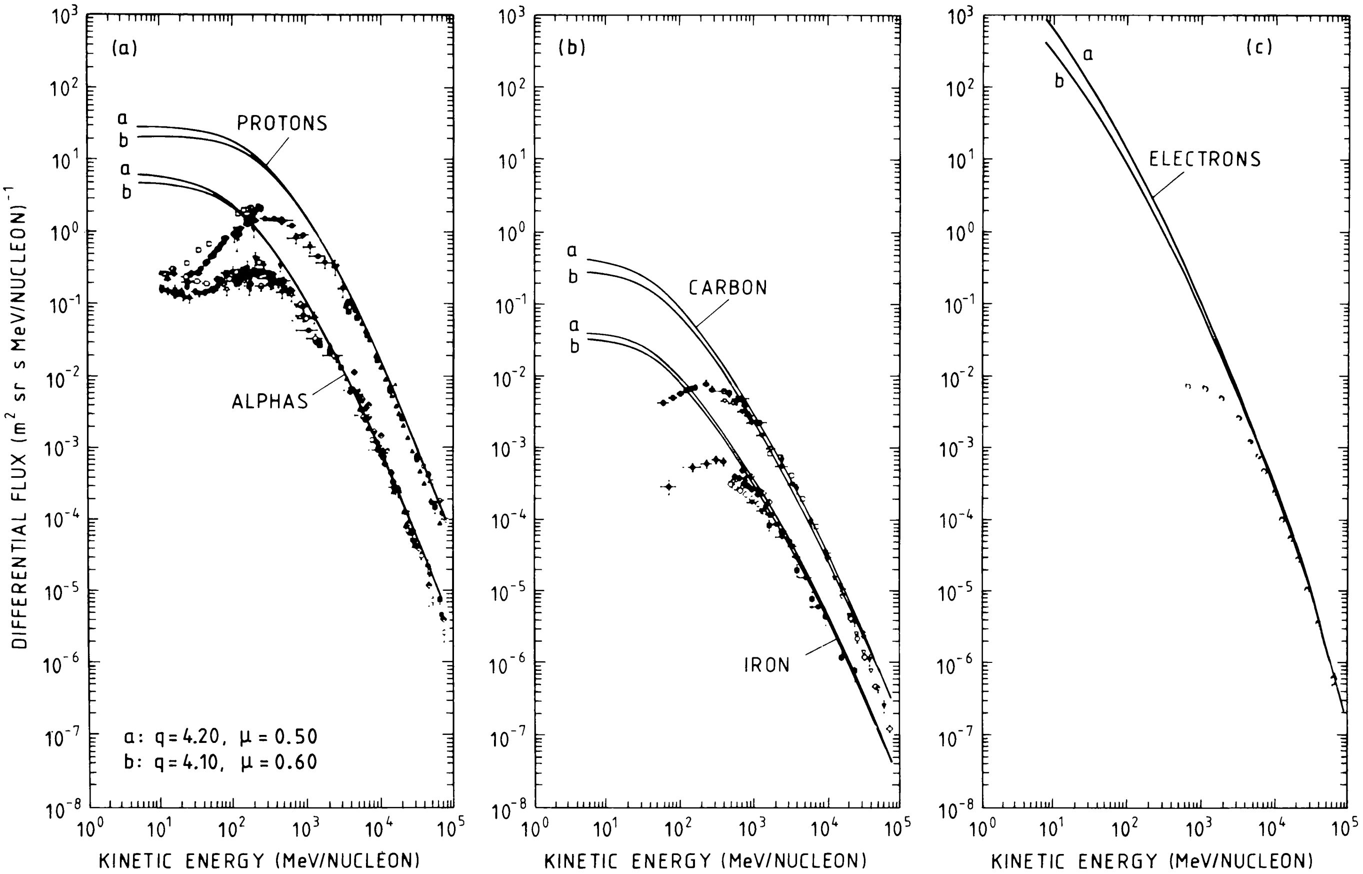

Fig. 1a–c. Cosmic ray energy spectra for **a** protons and alphas; **b** medium (C) and heavy (Fe) nuclei; and **c** electrons

Ormes and Freier (1978) and Ormes and Protheroe (1983) have considered the steady solution in a leaky-box model balancing the production rate of cosmic rays (characterized by a power law of $Q \sim T^{-\gamma}$ for the source strength) and the loss rate due to escape and interactions with the interstellar medium (nuclear spallation for ion species and synchrotron radiation for electrons). They have compared their numerical results to the observations from HEAO 3 and have examined the energy dependence for the source strength and the rigidity dependence of the particle escape time scale. However, as the ionization loss was not considered in their treatment, the cosmic ray energy spectra so derived would be appropriate only for particle energies above 2–3 GeV.

A different approach has been attempted by Lerche and Schlickeiser (1982); they considered analytical solutions of one-dimensional steady state production and propagation of cosmic rays in the galaxy. Instead of the leaky box approximation, both diffusive and convective effects have been taken into consideration. Of particular interest to us here is that ionization loss was found to dominate for energies below about 1 GeV and adiabatic deceleration due to galactic wind outflow is more significant in cooling the cosmic ray particles at higher energies. As the effect of ionization loss and Coulomb collision were incorporated in their formulation, the proton energy spectrum at low energies (<1 GeV) can be dealt with. Also they have shown that the cosmic ray pressure is practically constant at about $7\,10^{-13}$ dyne cm^{-2} for galactic heights smaller than several kpc. This means a phenomenal leaky-box model as depicted by Eq. (1) should be restricted to low galactic altitudes with $Z \lesssim 1$ kpc. In any event, one significant result of the work of Lerche and Schlickeiser (1982) concerns the flattening of the proton energy spectrum at low energies which is consistent with indirect observations of the interstellar spectrum (Fukui and Hayakawa, 1981). This feature is mainly caused by efficient ionization loss at this energy range and will be reproduced in our calculations.

Within the framework of the leaky box model, the escape time scale is determined by both convective loss and diffusion through the galaxy. At low energies, convective loss is most important; and at high energies, diffusive loss dominates. These two processes should be of comparable strength at kinetic energies $T \sim 2$ GeV for protons (see Jokipii and Higdon, 1979). Therefore, we use the following approximation for t_e in terms of the particle rigidity $R(=pc/ze)$:

$$t_e = \frac{t_0}{1+(R/R_0)^{\mu}} \tag{2}$$

with $t_0 = 3\,10^7$ yr, $R_0 = 1.7$ GV, and $\mu = 0.5$–0.7. The value of t_0 is chosen to agree with the cosmic ray residence times obtained from observations of the flux of (secondary) Be^{10} nuclei.

The $\frac{\partial p}{\partial t}$ term in Eq. (1) represents the momentum loss from Coulomb and ionizing collisions as well as nuclear interactions with the interstellar gas, and also the synchrotron and inverse Compton losses which are important for electrons. In our calculations, the expressions for the corresponding energy losses have been taken from Ginzburg and Syrovatskii (1964). The average number density of the interstellar hydrogen gas in the cosmic ray confinement volume is taken as $\langle n_{\rm H} \rangle = 0.27\,{\rm cm}^{-3}$ so that the integrated grammage is 6.61 g cm^{-2} at $T \cong 1$ GeV/nucleon (e.g. Garcia-Munoz et al., 1981). The energy density of electromagnetic waves (starlight plus cosmic black body rotation) in interstellar space near the galactic plane is taken to be 0.7 eV cm^{-3} (Allen, 1976) and the average interstellar magnetic field strength is assumed to be $\langle B \rangle \cong 3\,\mu$G.

To compare our calculated spectra with observational results we have transformed the distribution function $f(p)$ to the differential number density $U(T)$ and the differential intensity $j(T)$ by the relations

$$4\pi f p^2 dp = U dT, \quad \text{and} \quad j = \frac{1}{4\pi} vU, \tag{3}$$

where v is the particle speed and T the kinetic energy. The results for protons, alpha particles, carbon and iron nuclei and electrons obtained by starting the integration at $T = 100$ GeV/nucleon (normalized at $T = 60$ GeV/nucleon) for various values of μ and q are summarized in Fig. 1. Only the curves with $q + \mu = 4.7$ are shown as they provide the best fits to the observations. As can be seen in the figure, the solar wind modulation effect is significant only beginning at $T \leqq 5$ GeV/nucleon.

To examine the effect of energy losses on the energy spectra, in Fig. 2 the differential intensities calculated with only escape loss [i.e., $f = Q(p) t_e$] are compared with those obtained with all energy losses taken into account. For all species it can be seen that ionization loss begins to become the dominant effect only at energies <1 GeV/nucleon. At larger energies, these losses are less important and the energy spectra of protons and alpha particles are essentially determined by diffusive escape from the galaxy. On the other hand, synchrotron and inverse Compton losses are important for the cosmic ray electrons at high energies where a bend in the spectra occurs at ~ 30 GeV.

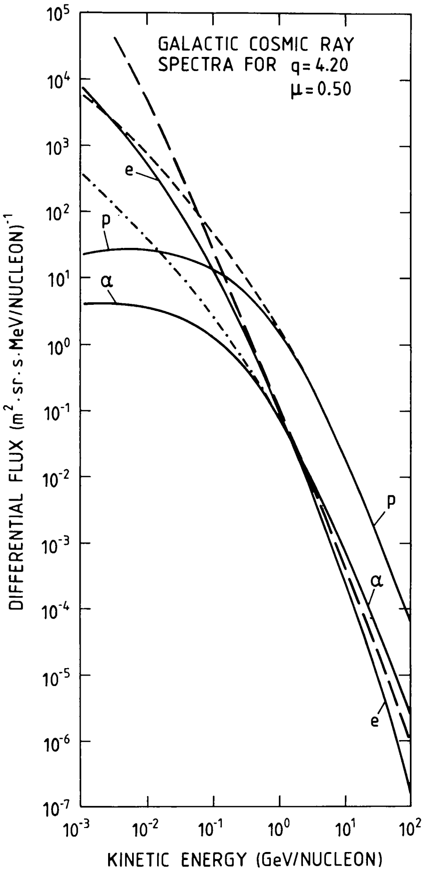

Fig. 2. Comparison of energy spectra of protons, alpha particles and electrons calculated by using full treatment of the energy loss and those with only escape loss (protons -----; alpha particles –·–·–; and electrons ––––)

From a comparison of the predicted electron spectra from their model calculations with observations over an energy range of 10–10^9 MeV, Protheroe and Wolfendale (1980) found that the best fit of the injection spectra could be given by $Q_e \propto E^{-2.1 \pm 0.1}$. This is in good agreement with our results (cases *a* and *b* in Fig. 1c) which provide a very good match to the interstellar cosmic ray electron spectrum determined by Webber et al. (1980) on the basis of radio data.

Reasonable fits for the energy spectra of proton, alpha, and heavies have been found using the same values of q and adopted in the electron calculations. As far as protons are concerned, the spectra are similar to those derived by Ormes and Protheroe (1983) – though we have extended the computation to much lower energies ($T \gtrsim 1$ MeV) in addition to the fact that ionization loss and Coulomb interaction have been accounted for in our model. Also note that the expression of the cosmic ray escape time (t_e) as given in Eq. (2) could be directly converted to their value of the escape length (λ_e) as derived from consideration of the recent HEAO 3 results (Bouffard et al., 1982). Ormes and Protheroe (1983) have remarked that, in part because of the solar modulation effect, the λ_e values at low energies (i.e., $T \lesssim a$ few GeV) are not well determined.

Further precaution can be raised about the validity of the leaky-box model at low energies. For example, the recent measurements of enhanced antiproton fluxes may be said to put constraints on the propagation process of the cosmic rays in the galaxy (Golden et al., 1979; Buffington et al., 1981; Protheroe, 1981). On the other hand, the enhancement of the $\bar{p}/p$ ratio may be traced to the production process of the antiprotons other than the trapping of the cosmic ray protons in the galaxy (Ginzburg and Ptuskin, 1981; Moraal and Axford, 1983). We would therefore defer consideration of these basic issues at the present moment and proceed with the more modest step of computing the total pressure and energy density of the cosmic rays.

Since the injection spectra at low energies $T \lesssim 1$ MeV/nucleon may differ from the power law of $Q(p) \propto p^{-q}$, the details of the differential fluxes in this low energy range in Fig. 1 may vary accordingly. However, assuming that the low energy particles do not contribute significantly to the total cosmic ray energy density (E_c) and pressure (P_c) in interstellar space, we can evaluate the corresponding cumulative values at energy T by the following expressions;

$$E_c = \int_0^T TU dT, \quad \text{and} \quad P_c = \tfrac{1}{3} \int_0^T \alpha TU dT, \tag{4}$$

where $\alpha = (T + 2T_0)/(T + T_0)$ with T_0 as the particle rest energy per nucleon. As an example, the numerical values of $E_c(\infty)$ and $P_c(\infty)$ for $\mu = 0.5$ and two different values of q are listed in Table 1.

The result $P_c(\infty) \cong 0.35$–0.47 eV cm^{-3} implies that the distance to the shock termination of the supersonic solar wind should not be less than about 100 AU (Axford, 1972). The variations of the cosmic ray energy densities and pressures as a function of kinetic energy are summarized in Fig. 3.

As stressed in the work of Lerche and Schlickeiser (1982), the proton energy spectrum should be quite flat at low energies. Similar result was also found in our present model calculations. It is illustrated quite clearly in Fig. 2 that the flattening of the energy spectrum of cosmic ray protons for energies less than a few GeV is mostly caused by ionization loss in the interstellar medium. This

Table 1[a]

	Protons	Alphas	Electrons	Total
$E_c(\infty)$	0.665 (0.850)	0.135 (0.170)	0.040 (0.070)	0.840 (1.09)
$P_c(\infty)$	0.274 (0.367)	0.058 (0.080)	0.013 (0.024)	0.345 (0.47)

[a] For $\mu = 0.5$ and unbracketed values are those calculated for $q = 4.2$ in units of eV cm^{-3}, and the bracketed ones are those for $q = 4.3$

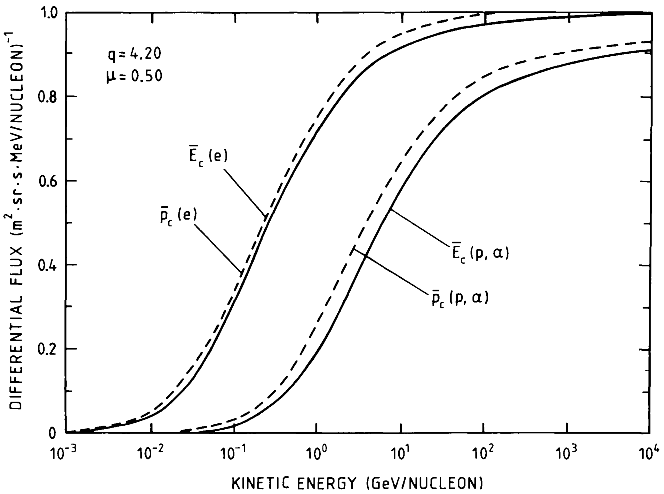

Fig. 3. Variations of the cosmic ray energy densities and pressures as a function of kinetic energy

means particles of MeV-energy could be quickly lost after production as a result of interaction with the turbulent and inhomogeneous interstellar matter (McCray and Snow, 1979; McKee and Ostriker, 1977). Consequently, the actual source strength and/or the relevant escape process would be masked. On the other hand, molecular observations of O Per in the Per OB2 association have been used to infer the operation of stochastic acceleration of MeV-cosmic rays via interaction of supernova remnant with the molecular cloud (Hartquist and Morfill, 1983). Supernova interaction with interstellar clouds thus should be important in shock acceleration of cosmic rays in general. The above discussions highlight several of the main uncertainties in the theoretical study of acceleration and propagation of low-energy cosmic rays. In our future work, attempts will be made to investigate the spatial and temporal effects related to shock acceleration in an inhomogeneous interstellar environment.

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References

Allen, C.W.: 1976, *Astrophysical Quantities*, Athlone Press, London, p. 268
Axford, W.I.: 1972, in *Solar Wind*, NASA SP-308, 609
Axford, W.I., Leer, E., Skadron, G.: 1977, *Proc. 15th Int. Cosmic Ray Conf. Plovdiv* **2**, 273
Axford, W.I.: 1981, *Proc. 17th Int. Cosmic Ray Conf. Paris* **12**, 155
Bell, A.R.: 1978a, *Monthly Notices Roy. Astron. Soc.* **182**, 147
Bell, A.R.: 1978b, *Monthly Notices Roy. Astron. Soc.* **182**, 443
Blandford, R.D., Ostriker, J.P.: 1978, *Astrophys. J.* **221**, 129
Blandford, R.D., Ostriker, J.P.: 1980, *Astrophys. J.* **237**, 793
Bogdan, T.J., Völk, H.J.: 1983, *Astron. Astrophys.* **112**, 129
Bouffard, M., Engelmann, J.J., Koch, L., Lund, N., Rasmussen, I.L., Peters, B., Soutoul, A.: 1982, *Astrophys. Space Sci.* **84**, 3
Buffington, A., Schindler, S.M., Pennypacker, C.R.: 1981, *Astrophys. J.* **248**, 1179
Field, G.B., Goldsmith, D.W., Habing, H.J.: 1969, *Astrophys. J. Letters* **155**, L49
Fukui, Y., Hayakawa, S.: 1981, *Proc. 17th Int. Cosmic Ray Conf. Paris* **2**, 226
Garcia-Munoz, M., Simpson, J.A., Wetel, J.P.: 1981, *Proc. 17th Int. Cosmic Ray Conf. Paris* **2**, 72
Ginzburg, V.L., Syrovatskii, S.I.: 1964, *The Origin of Cosmic Rays*, Pergamon Press, Oxford
Ginzburg, V.L., Ptuskin, V.S.: 1981, *Sov. Astron. Letters* 7 (5), 326
Golden, R.L., Horan, S., Mauger, B.G., Badhwar, D., Lacy, J.L., Stephens, S.A., Daniel, R.R., Zipse, J.E.: 1979, *Phys. Rev. Letters* **43**, 1196
Hartquist, T.W., Morfill, G.E.: 1983, *Astrophys. J.* **266**, 271
Johnson, H.E., Axford, W.I.: 1971, *Astrophys. J.* **165**, 381
Jokipii, J.R., Higdon, J.C.: 1979, *Astrophys. J.* **228**, 293
Koch, L., Perron, C., Goret, P., Cesarsky, C.J., Juliusson, E., Soutoul, A., Rasmussen, J.L.: 1981, *Proc. 17th Int. Cosmic Ray Conf. Paris* **2**, 18
Krimsky, G.F.: 1977, Dok. Akad. Nauk, SSR **234**, 1306
Lerche, I., Schlickeiser, R.: 1982, *Monthly Notices Roy. Astron. Soc.* **201**, 1041
McCray, R., Snow, T.P., Jr.: 1979, *Ann. Rev. Astron. Astrophys.* **17**, 213
Moraal, H., Axford, W.I.: 1983, *Astron. Astrophys.* **125**, 204
McKee, C.F., Ostriker, J.P.: 1977, *Astrophys. J.* **218**, 148
Ormes, J., Freier, P.: 1978, *Astrophys. J.* **222**, 471
Ormes, J.F., Protheroe, R.J.: 1983, *Astrophys. J.* **272**, 756
Protheroe, R.J.: 1981, *Astrophys. J.* **251**, 387
Protheroe, R.J., Wolfendale, A.W.: 1980, *Astron. Astrophys.* **92**, 175
Simon, M., Mathis, K.D.: 1983, *Proc. 18th Int. Cosmic Ray Conf. Bangalore* **2**, 215
Webber, W.R., Simpson, G.A., Cane, H.V.: 1980, *Astrophys. J.* **236**, 448