ICARUS 58, 109–120 (1984)

Some Dynamical Aspects of the Accretion of Uranus and Neptune: The Exchange of Orbital Angular Momentum with Planetesimals

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Received September 6, 1983; revised January 2, 1984

The final stage of the accretion of Uranus and Neptune is numerically investigated. The four Jovian planets are considered with Jupiter and Saturn assumed to have reached their present sizes, whereas Uranus and Neptune are taken with initial masses 0.2 of their present ones. Allowance is made for the orbital variation of the Jovian planets due to the exchange of angular momentum with interacting bodies ("planetesimals"). Two possible effects that may have contributed to the accretion of Uranus and Neptune are incorporated in our model: (1) an enlarged cross section for accretion of incoming planetesimals due to the presence of extended gaseous envelopes and/or circumplanetary swarms of bodies; and (2) intermediate protoplanets in mid-range orbits between the Jovian planets. Significant radial displacements are found for Uranus and Neptune during their accretion and scattering of planetesimals. The orbital angular momentum budgets of Neptune, Uranus, and Saturn turn out to be positive; i.e., they on average gain orbital angular momentum in their interactions with planetesimals and hence they are displaced outwardly. Instead, Jupiter as the main ejector of bodies loses orbital angular momentum so it moves sunward. The gravitational stirring of planetesimals caused by the introduction of intermediate protoplanets has the effect that additional solid matter is injected into the accretion zones of Uranus and Neptune. For moderate enlargements of the radius of the accretion cross section (2–4 times), the accretion time scale of Uranus and Neptune are found to be of a few 10^8 years and the initial amount of solid material required to form them of a few times their present masses. Given the crucial role played by the size of the accretion cross section, questions as to when Uranus and Neptune acquired their gaseous envelopes, when the envelopes collapsed onto the solid cores, and how massive they were are essential in defining the efficiency and time scale of accretion of the two outer Jovian planets.

1. INTRODUCTION

Differences in chemical composition and physical structure among the planets indicate differences in their accretion processes. For example, the terrestrial planets are generally assumed to be formed by the accretion of solid (rocky) material (e.g., Greenberg, 1979; Cox and Lewis, 1980; Wetherill, 1980: Cazenave *et al.*, 1982). On the other hand, the nearly solar composition of Jupiter and Saturn suggests that they accreted large amounts of gaseous hydrogen and helium. The two outermost Jovian planets, Uranus and Neptune—lacking a large gaseous component—are believed to be formed in a similar manner to the terrestrial planets, i.e., by the accretion of solid (a mixture of rocky and icy materials) bodies with only a small amount of gaseous components. Therefore, the accretion processes of the terrestrial planets and Uranus and Neptune are somewhat similar. However, there is also one basic difference: Uranus and Neptune are so massive that gravitational scattering of the planetesimals becomes one of the main factors, if not the dominant factor, in determining the time scale and details of the formation of these planets (Safronov, 1969, 1972).

Following this line of thinking, we have numerically investigated the simultaneous accretion and scattering of small bodies by proto-Uranus and proto-Neptune (Fernández and Ip, 1981, 1983). The general result is that, if the original population of plane-

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tesimals is concentrated in narrow accretion zones in the vicinity of the fixed orbits of the protoplanets, the accretion time scale is a few 10^8 years. Accompanying the gravitational accretion of the planetesimals, a large amount of small icy bodies will be ejected into interstellar space or be injected into long-period heliocentric orbits leading to the formation of the cometary Oort cloud. However, as has been pointed out by Safronov (1969), ejection of a large amount of planetesimals should also imply a sunward displacement of the accreting planet. From conservation of angular momentum we have

$$M_p \Delta(\sqrt{r}) = -(\sqrt{2} - 1)\sqrt{r}\,\Delta m_e \quad (1)$$

where M_p and Δm_e are the mass of the protoplanet and the amount of planetesimals ejected to escape orbitals, respectively. Assuming M_p to be constant, the radial displacement of the protoplanet as a total mass m_e is ejected, will be given by

$$\frac{r - r_0}{r_0} = \exp\left(-2(\sqrt{2} - 1)\frac{m_e}{M_p}\right) - 1. \quad (2)$$

Now, for $m_e \sim (0.1 - 1)M_p$ we have $\frac{r - r_0}{r_0} = -0.08$ to -0.56. Therefore, it seems likely that the orbital positions of Uranus and Neptune had been subjected to large changes during their accretion processes. As a first step toward a more realistic treatment of this interesting problem our previous numerical code was modified such that the proto-Uranus and proto-Neptune could meander around in the radial direction with their orbital radii determined by the exchange of angular momentum with interacting planetesimals (no collective effect involving the planets and the accretion disk is considered). One interesting result (which in hindsight was to be expected) that came out of this investigation is that the net radial motions of Uranus and Neptune are not always inward as predicted in Eq. (1) but rather outward. This is because there is a strong dynamical coupling between the accreting proto-Uranus and proto-Neptune, on the one hand, and Jupiter and Saturn, on the other, via the process of planetesimal scattering. In the following sections we shall discuss the details of the calculation and the assumptions used.

2. OUTLINE OF THE MODEL

The basic principle adopted before (Fernández and Ip, 1981) is employed again here, namely only close encounters between the Jovian planets and test planetesimals are considered by using Öpik's two-body formulation (Öpik, 1951). The computations start with proto-Uranus and proto-Neptune having masses 0.2 of their present ones, eccentricities of 0.05, inclinations of 0.02 rad and semimajor axes (for most of the computer runs) of $a_U = 20$ AU and $a_N = 30$ AU. As before, Jupiter and Saturn are included with their current masses and orbital parameters.

We have introduced some modifications in our program; for example:

(1) The restriction that the test planetesimals have to be in Uranus- or Neptune-crossing orbits has been removed. Instead, they are now spread out in the planetary disk with heliocentric distances ranging from 12 to 40 AU. The initial semimajor axes of the test planetesimals are randomly chosen within two possible distribution laws: a flat distribution ($n_1(a)$ = constant), or a decrease in the number of planetesimals (per unit of a) proportional to a^{-1} ($n_2(a) \propto a^{-1}$). Initial zero-inclination orbits with eccentricities of 0.05 are assigned to the test planetesimals.

(2) The orbits of the Jovian planets are allowed to vary following the exchange of angular momentum with the interacting bodies. Accordingly, after each interaction the new planetary orbit is computed from the conservation of momentum, where the test planetesimals are taken with masses m_c. For an initial mass M_{DISK} assumed to be equally distributed among 2000 test planetesimals, to each one will correspond a mass $m_c = M_{DISK}/2000$.

(3) A test planetesimal is assumed to be accreted by the interacting planet when it crosses the "accretion cross section," whose radius is

$$R_A = fR_G = fR(1 + 2GM_p/Ru^2)^{1/2} \quad (3)$$

where R_G is the gravitational radius of collision for the considered Jovian planet, R and M_p its radius and mass and u is the encounter velocity. $f(\geqq 1)$ is an enlargement factor that accounts for the possible presence in proto-Uranus and proto-Neptune of extended gaseous envelopes and/or circumplanetary swarms of bodies; effects that may contribute to trap incoming bodies as discussed below. For Jupiter and Saturn we adopt $f = 1$ on the assumption that they have already acquired their current inner structure.

(4) For some of the runs we have introduced two fictitious bodies with initial masses of $(1/3)M_\oplus$. They are located in mid-range orbits between Saturn and Uranus and between Uranus and Neptune with small eccentricities and inclinations ($e = 0.05$, $i = 0.02$ rad). We will refer to them as "intermediate protoplanets." They are allowed to accrete planetesimals but interactions with the Jovian planets—in case they become planet crossers—are neglected.

Some theoretical considerations give support to the idea of the early presence of this kind of intermediate protoplanets in the outer planetary region. For instance, Harris and Ward (1982) argue that the solar system is overstable in the sense that a greater number of bodies of planetary size could be located in the planetary region in a dynamically stable configuration. From the current obliquities of the rotation axes of the planets, Safronov (1966) has estimated the masses of the largest bodies that fell onto them. His computed largest colliding bodies with Uranus and Neptune turn out to be of the order of $1M_\oplus$ and $0.1M_\oplus$, respectively.

We should add a few words more to assumption (3). In previous investigations (Fernández and Ip, 1981, 1983) we found that the accretion time scale (t_a) of Uranus and Neptune is determined by the coupling between gravitational accretion and scattering of planetesimals as well as the total mass (M_T) of the planetesimals in the accretion zone. If M_T is too small, the mass of the protoplanet (M_P) will increase very slowly such that $t_a \gtrsim 4.5 \times 10^9$ years (Safronov, 1969). The phase of gravitational accretion is not reached in this case. If M_T is increased to large values, it does not mean that the protoplanet will acquire its present mass in a time interval arbitrarily short. This is simply because as the protoplanet grows to a mass sufficiently large to produce significant gravitational scattering the narrow accretion zone will be rapidly dispersed due to the continuous increase of the random velocity of the planetesimals. In turn, the gravitational capture will be quenched. Thus, in a way, we may say that the accretion time scale cannot be shorter than the corresponding dynamical time scale of gravitational scattering—which is in the order of 10^8 years (Fernández and Ip, 1981). Without the benefit of a narrow accretion zone at the beginnings, there will be a difficulty in initiating the stage of rapid growth. One possible way to bypass this problem is to assume an enhancement factor for the capture cross section. This could have come about in several ways (see Fig. 1). An optically thick accretion disk or ring system might have formed surrounding the accreting protoplanet (Harris and Kaula, 1975; Safronov and Ruskol, 1977; Harris, 1978). Inelastic collisions with disk particles might have helped to trap incoming planetesimals. Atmospheric gas drag has also been proposed as a capture mechanism (e.g., Pollack *et al.*, 1979; Nakazawa *et al.*, 1983). Interior models of Uranus and Neptune suggest that they consist of rocky–icy cores surrounded by hydrogen–helium envelopes of 1–$2M_\oplus$ (Hubbard and MacFarlane, 1980). A plausible scenario consistent with these interior models would be that Uranus and Neptune attracted signifi-

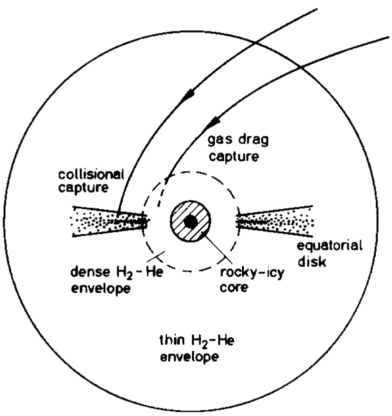

FIG. 1. A schematic view of how proto-Uranus and proto-Neptune could have captured bodies coming close to their solid cores, thus enhancing their accretion efficiencies: (1) In their final stage of accretion they might have been surrounded by extended gaseous envelopes making capture by gas drag possible. (2) Capture might have also occurred due to inelastic collisions with circumplanetary particles possibly distributed in a thin equational disk.

cant amounts of gaseous material around their accreting cores which remained as extended envelopes in hydrostatic equilibrium until the increase of the planetary mass caused their hydrodynamic collapse (Perri and Cameron, 1974; Mizuno, 1980). Should proto-Uranus and proto-Neptune have been surrounded by such extended gaseous envelopes, gas drag capture might have played an important role in their accretion histories.

Because the physical environment of accreting protoplanets has not been treated in any detail, it is difficult to quantify the enhancement factor f at this point. In our computations it will be considered as a free parameter.

Numerical results from 12 computer runs are shown in Table I. The initial mass M_{DISK} is expressed in units of the integrated current masses of Uranus and Neptune. Semimajor axes a are in AU. The final masses of Uranus and Neptune are in units of their current masses. The accretion time scales t_U and t_N (in 10^8 years) are defined as the time spans during which proto-Uranus and proto-Neptune accrete 90% of their masses. The residual mass in the outer planetary region and the mass ejected by Jupiter and Saturn are expressed as the fraction of the initial mass M_{DISK}. The value of M_{DISK} was established in such a way that the final masses of Uranus and Neptune matched in the best possible way their present masses. However, as the results show, that was not always accomplished.

3. ANGULAR MOMENTUM EXCHANGE

In this section we summarize the consequence of the dynamical coupling among the accreting Jovian planets via gravitational scattering of planetesimals across the solar system.

Because of the exchange of angular momentum with planetesimals, Uranus and Neptune (and to a lesser degree Jupiter and Saturn) are found to migrate from their original locations in the planetary disk. The extent to which they are radially displaced is a function of the mass M_{DISK} placed in the outer planetary region. The larger M_{DISK}, the larger the exchange of angular momentum and, hence, the larger the radial displacement of the Jovian planets. Because of their much larger masses, Jupiter and Saturn are found to experience only minor radial displacements. However, Uranus and Neptune may experience large displacements, as cases 1 and 2 of Table 1 show, where the final "Neptune" turns out to be closer to the sun than the final "Uranus."

In the Introduction we have considered briefly how an accreting planet moves radially inward because of the loss of its orbital angular momentum to the planetesimals ejected outward. The introduction of several perturbing planets greatly complicates the previous simple scheme. A typical example of what happens when several planets are considered in the dynamical evolution of a test body is shown in Fig. 2. The test body starts on a circular orbit at Neptune's heliocentric distance. For a certain time it random walks in the energy and an-

TABLE I

NUMERICAL RESULTS

Case	f	M_{DISK}	Distribution law of bodies	Other massive bodies	Final masses		Accretion time scale		Final semimajor axes and eccentricities								Residual mass	Mass ejected by Jupiter and Saturn	
					M_U	M_N	t_U	t_N	a_J	a_S	a_U	a_N	e_J	e_S	e_U	e_N		E_J	E_S
1	1	20	a^{-1} $12 < a < 40$	No	0.79	0.40	24.0	12.0	4.22	14.4	34.9	25.1	0.003	0.008	0.007	0.023	0.643	0.137	0.068
2	1	10	a^{-1} $12 < a < 40$	Yes	0.39	0.30	—	—	4.63	14.5	32.4	21.9	0.004	0.002	0.029	0.062	0.667	0.142	0.047
3	2	8	a^{-1} $12 < a < 40$	No	1.24	1.14	10.0	7.0	4.53	13.8	29.0	46.3	0.007	0.002	0.006	0.011	0.226	0.216	0.125
4	2	6	a^{-1} $12 < a < 40$	Yes	0.93	0.75	8.0	10.0	4.59	13.4	31.9	42.5	0.003	0.002	0.025	0.040	0.197	0.250	0.134
5	4	3	a^{-1} $12 < a < 40$	No	0.69	0.75	2.0	3.0	5.17	9.63	16.8	27.1	0.043	0.046	0.005	0.006	0.601	0.018	0.006
6	4	3	a^{-1} $15 < a < 40$	Yes	1.09	0.87	3.0	3.0	5.04	12.2	24.5	41.2	0.012	0.013	0.008	0.001	0.102	0.111	0.046
7	4	3	a^{-1} $12 < a < 40$	Yes	0.76	1.00	0.7	6.0	5.06	10.1	18.5	30.1	0.005	0.004	0.003	0.007	0.246	0.096	0.029
8	4	3	constant $12 < a < 40$	Yes	0.86	1.08	2.0	3.0	5.05	10.1	21.5	35.2	0.008	0.005	0.012	0.006	0.118	0.104	0.043
9	4	3	constant $12 < a < 40$	Yes	0.73	1.34	2.0	2.0	5.03	10.2	19.6	34.8	0.003	0.003	0.022	0.010	0.073	0.112	0.048
10	8	1.4	a^{-1} $12 < a < 40$	No	0.36	0.32	—	—	5.20	9.54	20.0	29.9	0.049	0.060	0.064	0.007	0.779	0.0	0.0
11	8	2	a^{-1} $12 < a < 35$	Yes	0.76	0.53	0.1	0.1	5.19	9.57	19.3	27.6	0.047	0.054	0.001	0.0005	0.531	0.009	0.002
12	8	2.4	a^{-1} $12 < a < 40$	Yes	0.59	0.49	0.3	0.4	5.20	9.56	20.1	27.9	0.048	0.059	0.011	0.006	0.517	0.004	0.002

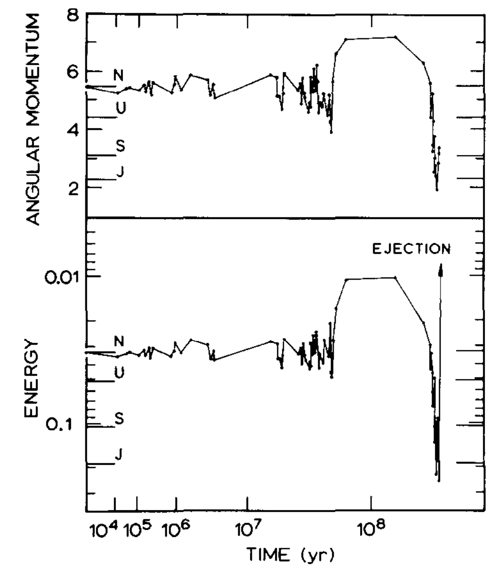

FIG. 2. Random walk of a body in the spaces of angular momentum and orbital energy until it is ejected by Jupiter's perturbation. The body starts on a circular and zero-inclination orbit at Neptune's distance. Its encounter velocity u increases with the number of close encounters with Neptune, so that the body eventually crosses the orbits of other giant planets and is perturbed by them. The angular momentum and energy are computed by adopting as units: $GM_\odot = 1$ and a in AU.

gular momentum spaces under the exclusive gravitational influence of Neptune. However, before being ejected by Neptune, the body falls under the gravitational influence of Uranus, Saturn, and finally of Jupiter that ejects the body out of the solar system. This analysis is similar to Weidenschilling's (1975) study on the probability that bodies scattered by Jupiter reach the region of the terrestrial planets before being ejected. For this purpose, he compares the smallest perihelion distance a body can be deflected with the minimum encounter velocity for ejection to be possible. The smallest perihelion distance $q_{\min}$ possible for a body after an encounter at velocity u with a planet of orbital radius unity is

$$q_{\min} = \frac{(1 - u)^2}{1 + 2u - u^2}. \qquad (4)$$

Bodies encountering Uranus at a velocity $u > 0.18$ can reach Saturn's orbit. For $u > 0.35$ they can reach Jupiter's orbit. For bodies encountering Neptune the corresponding figures are $u > 0.30$ and $u > 0.46$ (always taking the circular velocity at the distance of the planet as a unit). The minimum encounter velocity for a body to be ejected is $u = \sqrt{2} - 1 \cong 0.414$. Therefore, bodies under the gravitational influence of Uranus or Neptune are able to reach the region of Saturn (and Jupiter for those controlled by Uranus) before being ejected. A hypothetical planet would have to be at $r > 43$ AU for ejection of interacting bodies to be the predominant outcome as compared to scattering to the regions of Jupiter and Saturn.

As a result of the transfer of the body shown in Fig. 2 to the influence zones of the inner Jovian planets, it actually gives angular momentum to Neptune. This planet will thus experience an outward displacement, contrary to what was shown before for the case of a single planet. Jupiter, as the innermost and most massive Jovian planet, will be the main ejector of bodies. It will thus lose angular momentum and experience a sunward displacement. Table II shows the average angular momentum change of a body derived from two samples of test bodies starting in circular and zero-inclination orbits either at the distance of Uranus or that of Neptune. The quoted values result from adopting units of $GM_\odot = 1$ and r in AU. The angular momentum budget is negative for both samples, which means that Uranus and Neptune will on average gain angular momentum in their interactions with planetesimals.

TABLE II

AVERAGE ANGULAR MOMENTUM EXCHANGE OF A BODY OF UNIT MASS

Influence zone	$\overline{\Delta h}$
Uranus	$-4.9\ (\pm 2.4) \times 10^{-2}$
Neptune	$-7.8\ (\pm 0.25) \times 10^{-1}$

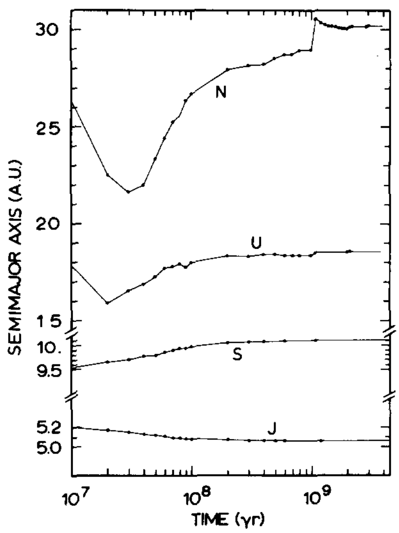

FIG. 3. Time variation of the semimajor axes of the four Jovian planets as a result of exchange of angular momentum with planetesimals. These results are taken from case 7. The initial semimajor axes are a_J = 5.203 AU, a_S = 9.54 AU, a_U = 20 AU and a_N = 30 AU.

The outward displacement of Uranus (Neptune) as a result of the removal from its accretion zone of an amount of mass m_r can be derived from Eq. (1), which leads to

$$\frac{r - r_0}{r_0} = \left(1 - \frac{\overline{\Delta h}}{\sqrt{r_0}} \frac{m_r}{M_p}\right)^2 - 1, \qquad (5)$$

where $\overline{\Delta h}$ is given in Table II for Uranus and Neptune.

A numerical example will help to illustrate the different outcomes whether we consider a single giant planet or all of them. Let us assume that Neptune removes from its accretion zone a mass equal to its own mass. This material will be finally ejected, should Neptune be the only giant planet influencing its dynamical evolution. Equation (2) would thus apply for $m_e = M_P$ leading to $(r - r_0)/r_0 = -0.56$ (sunward displacement). When the other giant planets are included a large proportion of the mass removed by Neptune will be transferred to their influence zones. Equation (5) will now apply for $m_r = M$ leading to $(r - r_0)/r_0 = +0.3$ (outward displacement). Note that the latter case gives a smaller displacement of Neptune. This is so because the transfer of bodies to the inner giant planets (gain of angular momentum for Neptune) is in part offset by the bodies ejected by Neptune (loss of angular momentum for Neptune).

Figure 3 shows the orbital displacement of the four Jovian planets as a result of the exchange of angular momentum with the scattered planetesimals obtained from one of the computed cases. This example shows that for the first 2–3 × 10^7 years Uranus and Neptune experience an inward displacement, since the interacting planetesimals in near-circular orbits on average gain energy and angular momentum following the so called Fermi acceleration mechanism (Arnold, 1965; Öpik, 1966). However, after many encounters the relative velocity u of the interacting planetesimals increases enough to allow them to become Saturn or Jupiter crossers. Such planetesimals are likely to be transferred to the influence zones of Jupiter and Saturn, usually with less angular momentum than they originally had. Uranus and Neptune will gain angular momentum and move outwards accordingly, as their graphs indicate for $t \gtrsim 3 \times 10^7$ years. Jupiter as the main ejector of bodies moves sunward. Saturn, like Uranus and Neptune, moves outward because it finally transfers a large fraction of the incoming planetesimals to Jupiter's influence zone. As mentioned, Jupiter and Saturn owing to their much larger masses experience smaller (though still significant) displacements than Uranus and Neptune.

Despite the large variations in the semimajor axes of Uranus and Neptune, their final orbital eccentricities are found to be in most cases very small ($e < 0.01$). Other effects, such as interactions with more massive bodies than those considered here or secular perturbations among the giant planets, are responsible for the present larger eccentricities of Uranus and Neptune (Ziglina and Safronov, 1976).

4. ACCRETION EFFICIENCY

As expected, the accretion efficiency is

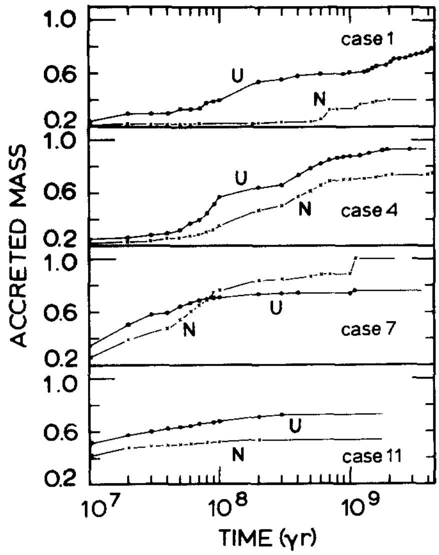

FIG. 4. Accretion mass rate of Uranus and Neptune for four cases described in Table I. Uranus and Neptune masses are expressed in terms of their current masses.

strongly dependent on the enlargement factor f adopted for the accretion cross section. A very low accretion efficiency is found for $f = 1$ ($R_A = R_G$). Accordingly, a very large initial mass M_{DISK} is required in order to form Uranus and Neptune (cases 1 and 2 of Table I). For instance, although a mass $M_{DISK} = 10$ is adopted for case 2, Uranus and Neptune fail to grow to masses comparable to their present ones. The time scale of accretion turns out to be longer than 10^9 years for $f = 1$ (Fig. 4, case 1). Both accretion efficiency and accretion rate greatly improve when f is increased to 2 and 4, as done for cases 3–9 (Fig. 4, cases 4 and 7). For $f = 4$ the accretion time scales of Uranus and Neptune decrease to $2–3 \times 10^8$ years, i.e., comparable to those obtained before (Fernández and Ip, 1981).

When the accretion cross section is artificially very large, e.g., $f = 8$ (cases 10–12), most of the planetesimals encountering Uranus and Neptune are rapidly accreted. When they sweep up all the bodies of their influence zones accretion comes to a halt, however. This is because the accumulated mass is not sufficiently large to start gravitational scattering and hence radial transgression to sweep up more planetesimals in other regions. A large amount of solid matter outside the accretion zones of Uranus and Neptune remains unaccreted and, therefore, these two planets fail to reach their current sizes (Fig. 4, case 11). This situation seems to lead to a different scenario in which many small planets (or asteroids) would form. A somewhat similar problem was discussed by Wetherill (1980). He argues that if the encounter velocity of planetesimals remains very low because of the predominance of energy dissipation mechanisms (or, as in our case, negligible scattering due to rapid accretion), then a large number of small planets tend to form rather than a few massive ones.

The introduction of intermediate protoplanets helps accretion since they stir planetesimals (originally in near-circular orbits outside the accretion zones of Uranus and Neptune) which in that way may become Uranus and Neptune crossers. This is clearly noted in the cases computed for $f = 4$. The intermediate protoplanets incorporated in cases 6–9 help to sweep up the residual planetesimals in the outer planetary region, whereas in case 5 (not including intermediate protoplanets) a large fraction of planetesimals remains at present. Significant displacements of the intermediate protoplanets occur due to the exchange of angular momentum with the interacting planetesimals. We may presume that the intermediate protoplanets eventually became Uranus or Neptune crossers and were subsequently accreted or ejected. This situation actually appears in several cases in which two hypothetical intermediate protoplanets were introduced but, due to our simple numerical treatment, we did not consider their interactions with Uranus or Neptune.

The inclusion of two intermediate protoplanets should be regarded as a very simplified approach to the real situation in which planetesimals of different sizes are stirred up by mutual perturbations. The effect of mutual perturbations among planetesimals has recently been treated by Greenberg *et al.* (1983). They find nonpower law distributions of planetesimal sizes with a large predominance of the smaller ones which might have favored accretion by keeping low relative volatiles among planetesimals. However, their model cannot be applied to the crucial latest accretion stage when proto-Uranus and proto-Neptune became massive enough to eject bodies from their influence zones.

The two a-distribution laws of planetesimals, $n_1(a)$ = constant and $n_2(a) \propto a^{-1}$, were applied for $f = 4$. The main difference to be noted is that Uranus tends to accrete a smaller amount of mass than Neptune when the distribution $n_1(a)$ is applied. This is because a larger fraction of solid material in Uranus' accretion zone is lost to the influence zones of Jupiter and Saturn. The amounts of mass accreted by Uranus and Neptune turn out to be better balanced when $n_2(a)$ is used, since the larger amount of mass available for Uranus in this case somewhat compensates for its greater losses.

Figure 5 summarizes several results derived from four computer cases using the distribution law $n_2(a)$. These results refer to final masses accreted by Uranus and Neptune and their displacements, and distribution of semimajor axes of the residual planetesimals. As seen, Uranus and Neptune do not always reach sizes comparable to their observed ones (e.g., cases 1 and 11). We note that larger radial displacements of Uranus and Neptune result when the amount of solid material allocated to the outer planetary region is larger (it increases from the bottom to the top).

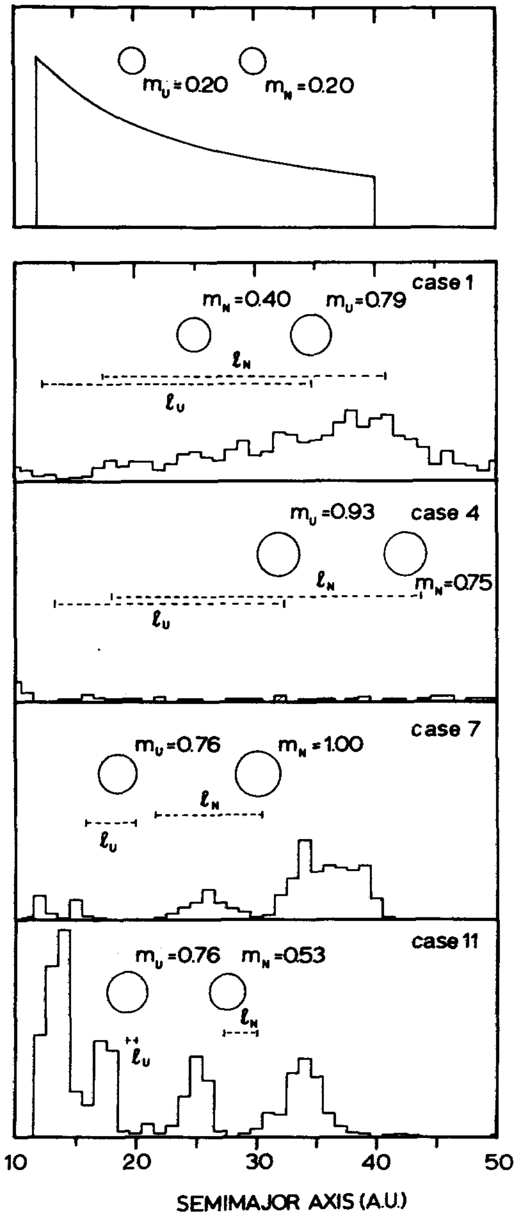

FIG. 5. The first diagram at the top shows the initial distribution of semimajor axes of planetesimals ($n(a) \propto a^{-1}$) and initial locations and sizes of Uranus and Neptune. These initial conditions are common to the four computer cases appearing below that show the distribution of semimajor axes of the residual planetesimals and masses and locations of Uranus and Neptune at present (t = 4.5 × 10^9 years). The lengths l_U and l_N represent the radial displacement of Uranus and Neptune throughout the solar system lifetime.

5. DISCUSSION AND SUMMARY

An important modification with respect to our previous numerical calculations (Fernández and Ip, 1981, 1983) was to relax the condition that all the test planetesimals cross either Uranus' orbit or Neptune's. Instead, we have distributed them throughout an ample range of heliocentric distances. We have also incorporated two effects not considered before, namely an enlarged accretion cross section for Uranus and Neptune and the gravitational stirring of disk planetesimals caused by intermediate protoplanets. For simplifying reasons several other effects had to be neglected as, for example, damping by interbody collisions or by nebular drag and long-range gravitational perturbations.

The neglect of damping effects may be to a large extent justified if it is accepted that a significant fraction of the solid mass was contained in large bodies little affected by damping. Secular perturbations may have an important long-term effect but for a system of bodies in which close planetary approaches are possible, as in our case, the strong perturbations there produced are likely to determine to a large extent the dynamical evolution of the system. A Monte Carlo study by Froeschlé and Rickman (1980) gives support to this view. Thus, they find that the diffusion speed of comets from intermediate-period orbits to short-period orbits is mainly determined by the strong close-range perturbations of Jupiter rather than by the more frequent but weak distant perturbations.

Because of constraints on computer time, samples of not more than 2000 test planetesimals could be computed. For initial masses M_{DISK} ranging between 1.4 and 20 $(M_U + M_N)$, that number corresponds to planetesimal masses ranging from $0.02M_\oplus$ to $0.3M_\oplus$, rather large as compared with the average mass of disk planetesimals although, as discussed before, of the order of the largest members expected within the size-distribution of planetesimals. It can happen that the planetesimal mass approaches that of the perturbing planet, in which case Öpik's two-body approach (that assumes a negligible mass for the planetesimal) is no longer suitable. Future studies should consider the mutual perturbations of two massive bodies coming to close approach as done, for instance, by Cox *et al.* (1978). Low-velocity encounters may also require a more careful treatment in the framework of the three-body problem (see, e.g., discussions by Cox *et al.* (1978) and Cazenave *et al.* (1982)).

Given the limitations of our numerical treatment and the complexity inherent to the accretion problem, we are perhaps still far from obtaining the definitive answers. Nevertheless, our study has highlighted a series of interesting possibilities that may contribute to the discussion of accretion of the outer planets and serve as guidelines for future work. In particular, we would like to mention the following:

(1) Uranus and Neptune might have undergone significant displacements during their final accretion stage that allowed them to sweep planetesimals throughout a large zone of the outer planetary disk. The extent to which migration proceeds is a function of the amount of solid mass scattered by the planets. An interesting result is that Neptune and Uranus (and Saturn) on average gain angular momentum in their gravitational interactions with a body, since it has a larger probability of being transferred to the inner giant planets. The three outer giant planets will thus experience an outward displacement. Jupiter, as the innermost giant planet and main ejector of bodies, loses angular momentum so it migrates sunward. This problem should be analyzed further considering, for instance, different radial distributions and orbital properties for the planetesimals as well as a more accurate treatment of the gravitational interactions along the lines mentioned above. See Safronov (1980). Damping effects—not discussed in our model—may play an important role in keeping low values of encounter velocities and, hence, in avoiding that planetesimals fall under the gravitational influence of two or more Jovian planets. However, it is likely that the largest planetesimals—probably with a heavy weight in the total exchange of angular mo-

mentum with the Jovian planets—were not so much affected by damping.

(2) The efficiency and time scales of accretion of Uranus and Neptune are strongly dependent on their accretion cross sections. For an enlargement factor $f = 1$ (accretion cross section equal to the gravitational cross section), very long accretion time scales (several 10^9 years) and a large initial mass (several tens of $M_U + M_N$) are required in order to form Uranus and Neptune (unless one assumes that matter was already concentrated in the accretion zones of Uranus and Neptune). Moderately enlarged accretion cross sections, due to the presence of gaseous envelopes and/or circumplanetary swarms of bodies, are found to greatly increase the efficiency of accretion and shorten the accretion time scales ($t_a \sim 10^8$ years). Very large accretion cross sections would have led to the rapid formation of several small planets for theirs would have efficiently swept all the matter of their accretion zones without appreciable radial displacements by angular momentum exchange. Mutual long-range gravitational interactions could perhaps have allowed their further accumulation into larger planets. However, the viability of this scenario was beyond the scope of our present study. In particular, questions as to when proto-Uranus and proto-Neptune acquired their gaseous envelopes, how extensive were they, and when they collapsed onto their rocky–icy cores are crucial in assessing the role of gas drag in the capture of planetesimals.

(3) The gravitational stirring of planetesimals due to the presence of intermediate protoplanets might have helped to clean the outer planetary region of residual solid matter. Planetesimals, initially outside of the Uranus and Neptune accretion zones, might have ended up as Uranus or Neptune crossers due to the scattering effect of these intermediate protoplanets. The stirring action by intermediate protoplanets together with the planetary displacement by angular momentum exchange may thus provide a solution to the long-standing problem of planetary accretion: namely, once the protoplanets swept away the matter in their accretion zones how did they continue accreting bodies from neighboring regions?

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