

參考  
月

甲、選擇題：共 8 題，每題 6 分，共 48 分。請用大寫字母 A, B, C, D 或 E 答題，並將答案依題號順序寫在答案卷上。皆單選。

1. Use the fact that  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$  to find the value of  $\lim_{x \rightarrow 0} \frac{2x}{\tan 7x}$ .  
 (A)  $\infty$  (B) 0 (C) 1 (D)  $\frac{2}{7}$  (E)  $\frac{7}{2}$

2. For what values of  $a, m$ , and  $b$  does the function  $f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \leq x \leq 2 \end{cases}$

satisfy the hypotheses of the Mean Value Theorem on the interval  $[0, 2]$ ?

(A)  $a = 3, m = 1, b = 4$  (B)  $a = 3, m = 2, b = 3$  (C)  $a = 3, m = 4, b = 1$

(D)  $a = 2, m = 1, b = 3$  (E) None of the above

3. Define  $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ . Find  $f'(0)$ .  
 (A) Does not exist (B) 1 (C) 2 (D)  $\frac{1}{2}$  (E) 0

4. Assume  $x = 2 \tan t, y = \sec^2 t - 1$ . Find an equation for the line tangent to the curve at the point where  $t = \frac{\pi}{4}$ .  
 (A)  $y = -x - 1$  (B)  $y = -x + 1$  (C)  $y = x$  (D)  $y = x + 1$  (E)  $y = x - 1$

5. Find the limit:  $\lim_{x \rightarrow \infty} \frac{1}{x \ln x} \int_1^x \ln t \, dt$   
 (A)  $\infty$  (B)  $-\infty$  (C) -1 (D) 0 (E) 1

6. Find the limit:  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^2}$ .  
 (A) 1 (B)  $\frac{1}{2}$  (C) 0 (D) 2 (E) Does not exist

7. Find the line integral of  $F = \langle 2xyz, x^2z, x^2y \rangle$  over any path from  $(0, 0, 0)$  to  $(1, 2, 3)$ ?  
 (A) 6 (B) 9 (C) 12 (D) 18 (E) None of the above

8. Find the surface area of the portion  $S$  of the cone  $z^2 = x^2 + y^2$ , where  $z \geq 0$ , contained within the cylinder  $y^2 + z^2 \leq 1$ .

(A) 0 (B)  $\pi$  (C)  $-\pi$  (D)  $2\pi$  (E)  $-2\pi$

參考用

乙、填充題：共 4 題，每題 8 分，共 32 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Evaluate the indefinite integral  $\int \frac{1}{1+e^x} dx$ .

Answer : \_\_\_\_\_

2. Find the points on the graph of  $z = 3x^2 - 4y^2$  at which the vector  $\mathbf{n} = \langle 3, 2, 2 \rangle$  is normal to the tangent plane.

Answer : \_\_\_\_\_

3. What is the value of the double integral  $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1} dx dy$ ?

Answer : \_\_\_\_\_

4. What is the largest value that the directional derivative of  $f(x, y, z) = xyz$  can have at the point  $(1, 1, 1)$ ?

Answer : \_\_\_\_\_

丙、計算、證明題：共 2 大題，每大題 10 分，共 20 分。須詳細寫出推論與演算過程，否則不予計分。

1. (a) Determine if the series  $\sum_{n=1}^{\infty} (-1)^n \cos \frac{1}{n}$  converges or diverges? (5 分)

(b) Determine if the improper integral  $\int_0^1 \frac{dx}{x - \sin x}$  converges or diverges? (5 分)

2. Find the maximum area of a rectangle inscribed in the ellipse  $\frac{x^2}{9} + \frac{y^2}{25} = 1$ . (10 分)